All The President's Money: Market Concentration, Oligarchs and Sanctions in Hybrid Regimes*

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Abstract

Hybrid regimes feature combinations of democratic and autocratic attributes. Their common element is a leader who cultivates a clientele of favored firms but also extracts income from them. We study the *economic* implications of this system in a general equilibrium economy where entrepreneurs can become clients by accepting informal rent-sharing contracts offered by the leader. The model derives the leader's incentives for creating economic distortions, and explains the emergence of oligarchs as a function of simple (institutional) constraints. We use the model to study the economic impacts of various sanctions, such as the freezing of the leader's or the oligarchs' assets, or the withholding of international transfers. Our results shed light on a number of recent and historical examples from hybrid regimes around the world.

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1 Introduction

Many countries today have political systems that are neither fully democratic nor fully autocratic, but “hybrid” regimes in between. For example, the 2022 index of the Freedom House characterizes 60 countries as “partially free” as opposed to “free” or “not free.”[1] This group includes six of the ten largest countries in the world, hence a sizeable share of the world’s population currently lives in this intermediate category.

While hybrid regimes used to be viewed as temporary structures present in countries that were undergoing a transition towards, or away from, democracy, they are now increasingly recognized as important systems in their own right (Levitsky and Way, 2002, 2010, 2020). A growing set of studies investigates the political processes that characterize these regimes - for example, how political leaders survive by exerting pressure on the media and controlling the flow of information to voters (Besley and Prat, 2006; Guriev and Treisman, 2020; Egorov and Sonin, 2020), by pitting rival groups against each other (Acemoglu et al., 2004), or through policy concessions and power sharing arrangements (Bueno De Mesquita et al., 2003; François et al., 2015; Bidner et al., 2015).

What has received much less attention are the *economic* implications of hybrid regimes. What is the impact of hybrid leaders on economic (as opposed to political) competition? What is the economic role of oligarchs and other entrepreneurs with ties to the party in power? How does public procurement work in such regimes, and how does its presence impact the rest of the economy? More broadly, how do the leader’s economic interests shape markets?

Studying these questions is all the more important given that hybrid regimes often invite sanctions from the international community. When considering the potential impact of these actions, policy evaluation is typically based on informal theoretical discussions. An economic model of hybrid regimes makes it possible to evaluate these arguments formally.

Our model of hybrid regimes focuses on the leader’s ability to extract income from firms operating in the economy, which appears to be a key distinguishing feature of such regimes. Dawisha (2015) summarizes the Russian case as follows:

> “Russian leaders needed “private” money [...] and they intended to get it, through more effective taxation but also through new arrangements with oligarchs that would provide more revenue for the state. [...] [This involved] oligarchs sharing their profits with the state and with Kremlin officeholders, including Putin, in return for a license to do business. Putin wanted the oligarchs to

[1] https://freedomhouse.org/report/freedom-world

> understand that they would have rents from these companies only as a reward for loyal state service." (p277).

This sharing of profits in return for a license to do business is a form of clientelism that we will refer to as *industry capture*. Industry capture may be contrasted with regulatory capture (Stigler, 1971), which is common in democracies, where firms have the power to influence policy makers (e.g., in the form of lobbying). In hybrid regimes, it is typically the leader who has the bargaining power, and (as in the above quote) uses it to dictate the terms of clientelistic arrangements. The widespread use of industry capture in hybrid regimes is likely due to a combination of factors, such as leaders' need to finance their political survival (e.g., by paying for propaganda), the weakness of institutional checks and balances that would prevent income extraction, and ultimately the selection of leaders who place high value on this income.

Importantly, industry capture by hybrid leaders faces constraints, which distinguishes these regimes from full-scale dictatorships. First, entrepreneurs cannot simply be forced to become a leader's clients and operate firms. There are economic opportunities that are not tied to the leader, and entrepreneurs can choose an occupation that is not in the leader's orbit. This gives rise to a *participation constraint*. Second, even if entrepreneurs agree to enter into a clientelistic contract with the leader, the leader's power to control and monitor them is not unlimited. The contract needs to make sure that clients do not have an incentive to abscond with or hide their profits instead of sharing it with the leader. This creates an *enforcement constraint*.

Our model combines these observations about hybrid leaders with a general equilibrium economy. The setup, which we present in Section 2, features a continuum of agents who establish firms, work, and consume in a multisector economy with monopolistic competition akin to Dixit and Stiglitz (1977). We do not model the politics of how the leader gets elected or how he acquires specific powers. Instead, we simply assume that the leader has the ability to capture industries: (i) he determines who can become an entrepreneur and establish firms, and (ii) he extracts income from these entrepreneurs in return for the right to operate. These actions are subject to the participation and enforcement constraints described above. When deciding on firm entry and income extraction, the leader considers both his income and social welfare.

Section 3 derives the core implications of this model. In a benchmark with a purely welfare-maximizing leader, the clientelistic system replicates a free-entry economy: the leader chooses the same number of firms as the market would, and extracts no income. When his value of extracted income is positive, however, the leader chooses to restrict entry in order to increase profits. The reason for this is that, in equilibrium, increasing market concentration is the only way to create the extra income that can be extracted. This in turn leads to welfare losses. An immediate implication is that checks and balances that limit the opportunities for industry capture, or improving the political selection of leaders with low value for extracting income, lead to more competitive markets and higher social welfare.

As the value of extracted income grows (which may reflect, e.g., weaker institutions or worse political selection), the distortions from industry capture become more severe. However, the leader's limited power over his clients endogenously mitigates this effect. As clients' profits increase, their incentive to abscond also rises, and this can only be offset by providing them with rents. A leader with low power over his clients or a moderate value of extracted income finds this too costly, and prefers to extract less.

As the leader's power or the value of extracted income grows beyond this range, a qualitatively different regime emerges. Entry restrictions and profits rise, but the latter is now shared with clients who receive rents - they become "oligarchs." Thus, oligarchs are associated with particularly severe economic distortions and welfare losses. Note however that oligarchs with large rents are a symptom, not a cause of these negative effects: they reflect a leader who values his income enough that he is willing to extract more even if this requires sharing it with his clients. In fact, for such a leader the need to deal with oligarchs acts as a moderating force on his actions. As the leader's power rises, this moderating force weakens. Hybrid regimes with generally strong leaders (i.e., leaders with more power over their clients across many industries) can eschew rents *and* are particularly harmful for competition and welfare.

We also study settings where a leader has heterogenous power over clients in different industries. For example, the leader is likely to have more power over a real estate developer whose business is heavily dependent on government licenses and regulations, than over a technology company producing for the international market. We show that competition in an industry benefits (in a second-best sense) from a leader with extensive powers in *other* industries. Intuitively, if the leader's power in an industry goes up, he will choose to extract more profit from that industry. With diminishing marginal utility of income, this reduces his incentive to extract income from other industries, which causes him to allow more competition elsewhere in the economy.

At the same time, we show that the leader's power in other industries gives him an incentive to impose entry restrictions even in an industry from which he cannot extract income. This is due to general equilibrium effects: restricting entry in this industry raises profits in all other industries, allowing the leader to extract more income there. As long as such spillover effects are present, the free-entry outcome in *any* industry can only be achieved by eliminating the leader's power over *all* industries.

In Section 4, we use our model of the economy under a hybrid leader to study various sanctions - a tool of international policy making that has seen a sharp increase since the end of the Cold War. The overarching message from our model is clear: the impact of sanctions depends on how the various actors (firms, oligarchs, and the leader) will respond to them as they pursue their objectives. Broad economic sanctions can hurt consumers and social welfare without any offsetting benefits. Sanctions that directly target the leader or the oligarchs (sometimes called "smart" sanctions in the literature) can lower the leader's income but give rise to undesirable policy responses. For example, a sanction that freezes the leader's assets will increase his marginal utility of income. The leader responds by extracting more income from his clients, which requires higher profits and more entry restrictions. Freezing oligarchs' assets has essentially the same consequences, because an oligarch with reduced income will need to be compensated with higher rents by the leader in order to keep him from absconding. To be successful at lowering the leader's income without imposing large welfare losses, sanctions need to be even "smarter" and must take into account the *mechanism* through which the leader extracts his income.

In Section 5, we study another area of government activity where hybrid leaders play a key economic role: public goods provision through procurement. We show that the importance of public procurement in hybrid regimes is due precisely to the constraints that such regimes impose on the leader. To study this, we consider a world where the government hires private companies to provide public goods, and pays for them using tax revenues earmarked for this purpose. The leader cannot simply divert tax revenues into his private income. However, he can use his powers over the public procurement process to achieve essentially the same goal. In particular, the leader will purchase public goods from his client-entrepreneurs at an inflated price, and then extract the resulting extra profit. This provides an explanation for why, empirically, client-entrepreneurs are often clustered in industries involved in public procurement.

In this context too, successful sanctions are those that do more than just limit resources. For example, limiting international transfers will both reduce public good spending and incentivize the leader to restrict competition. By contrast, policies that target the income extraction mechanism by limiting overpricing in public procurement can lower the leader's income while *increasing* welfare.

We study several extensions of our model in Section 6, investigating the incentives to innovate in hybrid regimes, the economic effects of a leader who favors entrepreneurs over workers, and rent-seeking among oligarchs.

Finally, in Section 7 we use our results to interpret a number of examples of hybrid regimes around the world, including Russian oligarchs, privatization in Latin America, the

"License Raj" in India, crony firms in North Africa, and the use of European Union funds in Hungary.

Related literature. As discussed above, most existing models focus on the politics of how hybrid leaders acquire and keep power, rather than the economic implications of these regimes.[2] An important exception is Acemoglu (2008) who studies the long-run dynamics of elite formation in an oligarchy vs a democracy. In his model, workers who enter the elite (by establishing firms) cement their status by introducing entry barriers through collective choice.[3] The elite in our model is more centralized: it features a fixed leader and his hand-picked cronies, and we focus on the interaction of these agents through clientelism and public procurement. Our model features multiple sectors, making it possible to study spillovers across industries, and we also use it to study the impact of sanctions against hybrid regimes.

More generally, our paper is related to a literature investigating the general equilibrium effects of lobbying (e.g., Grossman and Helpman (1994); Bombardini and Trebbi (2012); Huneeus and Kim (2021)).[4] While lobbying is a key channel linking firms and politicians in established democracies, its relative importance is lower in hybrid regimes where the bargaining power rests squarely with the leader. This is particularly apparent in the channels that are the focus of our paper: clientelism and the public procurement process. As we show, the implications of these channels can differ from those of lobbying models, particularly in general equilibrium.

Our paper is also related to a growing stream of papers on the causes and consequences of increasing market concentration. Focusing mostly on the US and other developed countries, this literature distinguishes between "good" market concentration driven by changes in preferences and technology, and "bad" market concentration driven by increasing entry barriers (see e.g. Autor et al., 2017; Haskel and Westlake, 2018; Covarrubias et al., 2020). Our model describes a novel source of the latter type of market concentration in hybrid regimes.

Finally, because income extraction by the leader is a form of corruption, our paper is related to the massive corruption literature. Shleifer and Vishny (1993)'s seminal observation that corrupt officials have an incentive to create scarcity operates in our model through entry restrictions, and indeed, empirically, corruption is often positively correlated with entry restrictions and market concentration (Ades and Di Tella, 1999; Djankov et al., 2002).

[2] A larger empirical literature studies the economic role of political connections in nondemocracies / hybrid regimes - see, e.g., Fisman (2001), González and Prem (2020), and Szeidl and Szűcs (2021).

[3] See also Shadmehr (2019), where capitalists favor regulations (capital controls) because this can lower the likelihood of regime change through a revolution.

[4] There is also a large literature on political influence in a partial equilibrium economy. Recent work includes Cowgill et al. (2023); Akcigit et al. (2023), and Callander et al. (2022). See also Shleifer and Vishny (1994) and Boycko et al. (1996) on state-owned firms.

However, few papers have combined this aspect of corruption with an explicit economic model. One exception is Aidt and Dutta (2008), who model the economic implications of corruption in a democracy, focusing on how economic growth affects a corrupt politician's incentive to maintain entry barriers.[5] Our contribution is to provide an explicit model of a type of corruption that is particularly relevant in hybrid political regimes, and embed it in a general equilibrium model in order to analyze its economic implications. This allows us to study how specific features of corruption affect entry restrictions, how these translate into economic outcomes (such as competition, product diversity, and social welfare), and how international policies or sanctions may affect these regimes.

2 Setup

There is a continuum of agents and a leader. Agents work, consume and produce in a general equilibrium economy with monopolistic competition akin to Dixit and Stiglitz (1977). We do not model the politics of how the leader gets elected or how he acquires specific powers.[6] Instead, we simply assume that the leader has powers that, empirically, seem fundamental to hybrid regimes. Specifically, the leader has the power to capture industries: (i) he determines who can become an entrepreneur and establish firms, and (ii) he extracts income from these entrepreneurs in return for the right to operate. When deciding on firm entry and income extraction, the leader considers both his income and social welfare. As we shall see, one of the key characteristics that makes this regime hybrid rather than, e.g., a totalitarian dictatorship, is that industry capture by the leader must respect entrepreneurs' participation and enforcement constraints.

In the rest of the section we introduce the details of this environment.

2.1 Consumption, production, and the free-entry equilibrium

There is a unit mass of ex-ante identical agents, each endowed with a unit of labor. Agents have Cobb-Douglas utility across the products of $J+1$ industries indexed $j = 0, ..., J$, with a CES aggregator across varieties produced within an industry. The utility of each agent is

$$\prod_j Q_j^{\hat{\beta}_j} \tag{1}$$

[5] See also Bliss and Di Tella (1997) and Emerson (2006).

[6] Nor do we model *why* the regime is hybrid rather than a full-scale dictatorship. Levitsky and Way (2020) discuss various forces that could account for this.

where $\sum_j \hat{\beta}_j = 1$, and

$$Q_j = \left[\int_0^{\Omega_j} q_j(\omega)^{\frac{1}{\mu}} \, d\omega \right]^{\mu}$$

is the quantity index of industry $j > 0$. The term $q_j(\omega)$ is the quantity of variety ω while Ω_j is the mass of varieties produced in industry $j > 0$. The coefficient of the elasticity of substitution, $\mu > 1$, is constant across industries. Industry $j = 0$ produces a homogeneous good, therefore Q_0 is the quantity consumed of the good in industry 0. It will be convenient to use the following notation: $\beta_j \equiv \hat{\beta}_j \frac{\mu-1}{\mu}$ and $\bar{\beta} \equiv \sum_{j>0} \beta_j = \frac{\mu-1}{\mu} - \beta_0$.

All $J+1$ industries use labor as the only input. Agents use some of their labor as workers and some as entrepreneurs. We do not restrict this choice to be binary: agents can divide their unit of labor endowment between the two occupations (for example, they can spend some of their time on entrepreneurial tasks involved in setting up and managing a firm, and the rest on production tasks). Worker labor is fully mobile across industries, and we normalize its wage to 1. Entrepreneurial labor is industry-specific, i.e., each agent can only work as an entrepreneur in a specific industry.[7]

In each industry $j > 0$, production has both a fixed cost f_j and a variable cost c_j in terms of labor. Specifically, producing quantity $q_j(\omega)$ of variety ω requires f_j units of labor provided by entrepreneurs to set up a firm, and $c_j q_j(\omega)$ units of labor provided by workers.

The homogeneous good in industry $j = 0$ is produced with unit variable input requirement, $c_0 = 1$, and no fixed cost, $f_0 = 0$. We assume free entry in this industry, so that this good is provided in perfectly elastic quantity for a price $p_0 = 1$. This is the numeraire good.[8]

Each firm produces one variety, choosing its price $p_j(\omega)$ to maximize profit $\pi_j(\omega) \equiv q_j(\omega)(p_j(\omega) - c_j)$. Suppose agent $i \in [0, 1]$ has income $Y(i)$, and let Y denote aggregate income. By standard arguments[9] we obtain that equilibrium prices and quantities will be the same across varieties within an industry (so we drop the index ω from now on). Specifically,

[7] This assumption is necessary to allow heterogeneity across industries in equilibrium.

[8] The presence of a numeraire industry will only play a role in the analysis of public procurement in section 5. All other results also hold setting $\hat{\beta}_0 = 0$.

[9] See Appendix A.

for all $j > 0$, we get

$$p_j = \mu c_j \tag{2}$$

$$q_j(i) = \frac{\beta_j}{\Omega_j(\mu-1)c_j} Y(i) \tag{3}$$

$$Q_j\left(i\right) = \frac{\beta_j \Omega_j^{\mu-1}}{(\mu-1)c_j} Y\left(i\right) \tag{4}$$

$$\pi_j = \frac{\beta_j}{\Omega_j} Y \tag{5}$$

where $q_j(i)$ and $Q_j(i)$ denote individual i's consumed quantity and quantity index, respectively. Note that π_j does not include the "fixed cost" of entrepreneurs' labor f_j. For the numeraire industry, $Q_0\left(i\right) = \hat{\beta}_0 Y\left(i\right)$ and $\pi_0 = 0$.

Income $Y(i)$ comes from two sources: labor and profits. Workers earn wages (normalized to 1 per unit of labor), while entrepreneurs receive a share of the firm's profit equal to π_j/f_j per unit of labor. Given Ω_j, total income of workers is $1 - \sum_j \Omega_j f_j$ and total income of entrepreneurs is $\sum_j \Omega_j \pi_j$, so that

$$Y = 1 + \sum_{j=1}^{J} \Omega_j(\pi_j - f_j). \tag{6}$$

To close the model, we need to determine the number of entrepreneurs and hence the number of firms. In our model, this will be set by the leader subject to various constraints. As a benchmark, assume for a moment that there is free entry into entrepreneurship. In this case, equilibrium requires that agents be indifferent between using their labor as workers or as entrepreneurs. Thus, it must be that

$$\frac{\pi_j}{f_j} = 1. \tag{7}$$

Using (5), (6), and (7), we get that in this *free-entry equilibrium*, the number of firms in industry $j > 0$ is given by

$$\Omega_j = \Omega_j^{FE} \equiv \frac{\beta_j}{f_j}.$$

From now on, when it does not cause any confusion, we use the index j for industries $j > 0$, excluding the constant-returns-to-scale industry $j = 0$.

2.2 The leader-client contract

To model the economy under a hybrid political system, we assume that there is a leader who has the power to determine who can become an entrepreneur. We model the leader as a unitary actor, but this might represent a small inner circle, such as a strongman chief executive and his family members, close allies, or the upper echelons of his party.

In return for the right to operate firms, entrepreneurs must hand over part of their profit to the leader. In other words, the only way to become an entrepreneur is to enter into a clientelistic contract. These contracts will specify the amount of profit that entrepreneurs in a firm can keep, w_j, with the remaining $\pi_j - w_j$ handed over to the leader.

In our model, the clientelistic contract is subject to two fundamental constraints. The first is that agents cannot be forced to become clients against their will: they can always use their labor as workers, and will only become entrepreneurs if this is worth it for them. Since workers earn a wage of 1 per unit of labor, this *participation constraint (PC)* is

$$w_j \geq f_j \tag{8}$$

The second constraint is that the enforcement of the contract is limited, in the spirit of Kehoe and Levine (1993). Specifically, we assume that clients can abscond with a share $(1 - \phi_j)$ of the profits. For example, entrepreneurs could move their profit abroad or to the shadow economy (Johnson et al., 1997), or shield it from the leader through defensive ownership structures (Earle et al., 2022).[10] For now we assume that ϕ_j is fixed - in an extension, we will consider the possibility that clients increase it by exerting effort. Absconding clients forfeit their payment w_j, but they cannot be subjected to any other punishment by the leader. In this sense, the hybrid leader's clients benefit from limited enforcement. This gives rise to an *enforcement constraint* (EC):

$$w_j \geq (1 - \phi_j)\pi_j. \tag{9}$$

Limited enforcement creates an "efficiency wage" role for w_j in incentivizing clients not to abscond with their profits. Throughout, we assume that $\phi_j > 0$ for at least some j.

The parameter ϕ_j can be interpreted as the leader's power over his clients. More powerful leaders need to give up less of their income in order to incentivize their clients. Power may arise from the leader's personal ties to clients: for example, a close social contact or party

[10] We implicitly assume that the (out-of-equilibrium) decision to abscond with the firm's profit would be made jointly by all the entrepreneurs working in a firm. This ignores potential collective action problems between clients (which a sophisticated leader might be able to exploit). Studying such problems may be an interesting avenue for future research.

member may find it harder to abscond with his profits than an entrepreneur at arm's length from the leader. Power may also arise from the nature of the industry's activities. For instance, the leader may have more power over an oil company or a property developer whose business heavily depends on government licenses and regulations, compared to a technology firm producing for the international market who might even be able to relocate its business to another country at some cost. Below, we study the impact of changes, as well as heterogeneity, in ϕ_j on clientelistic contracts and the economy.[11]

While stylized, we believe the participation and enforcement constraints capture important features of hybrid regimes that distinguish them from either totalitarian dictatorships or established democracies. Although clientelism can also be pervasive in totalitarian systems, a dictator's power over his clients tends to be quite extensive, e.g., he may simply expropriate a firm's profit and throw managers in jail if they stand in the way. Thus, we do not expect the enforcement constraint (and perhaps not even the participation constraint) to matter. In established democracies, clientelism is relatively uncommon. "Profit sharing" between firms and the government takes place through legally codified channels, such as the tax system. Because they are backed by the legal system, such profit sharing contracts are easy to enforce. Thus, again, we do not expect the enforcement constraint to play an important role.

2.3 The leader's problem

In a hybrid political regime, the leader chooses the number of firms in each industry,[12] $\Omega = (\Omega_1, ..., \Omega_J)$, and the clients' payments $w = (w_1, ..., w_J)$ to maximize a combination of social welfare W (the sum of all agents' utility) and the income he obtains from his clients, $Y_L \equiv \sum_j \Omega_j(\pi_j - w_j)$.

In some hybrid regimes, the leader values his income Y_L because it is essential to maintain his power by financing political propaganda or vote-buying. In others, this income represents the funds the leader can keep out of the public eye and use for his family's personal consumption. Regardless of the deeper determinants of his preferences, we simply take it as given that Y_L is valued by the leader. A key parameter in our analysis is a weight, denoted

[11] In a dynamic version of our setup, (9) could be rationalized by assuming that the client can abscond with and sell a share of the firm's final products, while the leader can exclude the absconding client from future profit sharing contracts. Then the client has to be paid above the net expected gain from such self-dealing. (See Kehoe and Levine (1993) and Rampini and Viswanathan (2010) for similar dynamic arguments.) Alternatively, ϕ_j can also be interpreted as the leader's bargaining power relative to his clients. If profit sharing between the leader and his clients takes place through Nash bargaining with bargaining weights ϕ_j for the leader and $1 - \phi_j$ for clients, we again obtain expression (9).

[12] As will be clear below, some industries could be excluded from the leader's choice set without affecting the analysis. These industries would have free entry, and their profits would just cover the fixed costs f.

with λ, that the leader places on this income relative to social welfare. Strong democratic institutions with robust checks and balances limit the leader's value from extracted income, resulting in a small λ. As institutions weaken, λ is likely to grow. We do not model where λ comes from - we use it to summarize the exogenous, institutional determinants of leaders' value from extracting income from the economy. This can be contrasted with the endogenous determinants of income extraction that we study below.

Total income of all entrepreneurs is $\sum_j \Omega_j w_j$, so total income of all agents in the economy (without the leader) is

$$\int_0^1 Y(i)di = 1 + \sum_j \Omega_j (w_j - f_j). \tag{10}$$

Social welfare is

$$W \equiv \int_0^1 \prod_j Q_j(i)^{\hat{\beta}_j} di,$$

and using (4) and (10) this can be written as

$$W(\Omega, w) = \hat{\beta}_0^{\hat{\beta}_0} \int_0^1 Y(i)di \prod_j \left(\frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j} \right)^{\hat{\beta}_j} = \hat{\beta}_0^{\hat{\beta}_0} \left(1 + \sum_j \Omega_j (w_j - f_j) \right) \prod_j \left(\frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j} \right)^{\hat{\beta}_j}.$$

For tractability, we specify the leader's objective as Cobb-Douglas: $Y_L(\Omega, w)^{\lambda} W(\Omega, w)^{1-\lambda}$. Thus, taking logs and dropping the constants, the leader solves

$$\max_{\Omega, w} \lambda \ln \left(\sum_j \Omega_j (\pi_j - w_j) \right) + (1-\lambda) \left[\ln \left(1 + \sum_j \Omega_j (w_j - f_j) \right) + \sum_j \beta_j \mu \left(\ln \Omega_j - \frac{\ln c_j}{\mu - 1} \right) \right] \tag{11}$$

subject to (5),(6),(8), and (9).[13]

The choice of Ω and w determines the level of competition, prices and quantities in each industry, as well as all agents' income in the economy (as reflected in the constraints (5) and (6)). When the value of extracted income (λ) is zero, the objective function in (11) nests a welfare maximizing leader.

[13] Because we model the economy in general equilibrium, how the leader spends this income affects prices, quantities, and profits. To keep things simple, this formulation assumes that the leader allocates his income across varieties and industries in the same proportion as every other agent. Hence, for any Ω and w, the equilibrium in this economy is still described by the system (2)-(6), with total spending given by (6). (Studying a leader who overspends in certain sectors would be a simple extension of our analysis.)

3 Regulation and income extraction in a hybrid regime

3.1 A benchmark: the welfare maximizing leader

The following proposition describes the leader's choice and the economy in the special case of $\lambda = 0$, i.e., a welfare maximizing leader. (All proofs are in the Appendix.)

Proposition 1 *When* $\lambda = 0$, *we have*

$$\pi_j = f_j$$
$$\Omega_j = \Omega_j^{FE}.$$

For a welfare maximizing leader, we obtain the same solution as the free-entry equilibrium described in Section 2.1. Here, entrepreneurs keep all their profits, and the leader allows the same number of firms to operate as would emerge in equilibrium if entry into entrepreneurship was free. Clientelism plays no role: in effect, all firms remain independent from the leader.

3.2 Income extraction and entry barriers

When the value of extracted income is positive ($\lambda > 0$), the leader may choose to restrict the number of firms below its free-entry level. Intuitively, in the free-entry solution profits are only large enough to cover the fixed costs f_j of operation. Since entrepreneurs cannot be forced to participate in the clientelistic contract, in order to extract income the leader must first raise profits. This is accomplished by restricting entry. Proposition 1 implies that this will reduce welfare.

From the clients' perspective, once some of their profit is extracted by the leader, their firms only stay in business *because* the leader restricts entry and creates enough profit to cover both the fixed costs and the extracted share. In this sense, incumbent firms become dependent on the regulation of entry, and hence the leader, for their survival.

The following proposition characterizes the different types of hybrid regimes that arise in equilibrium as a function of the parameters. Here we focus on the case when the leader's power is the same across industries $\phi_j = \phi$ and study the case of heterogenous powers $\phi_j \neq \phi_k$ in Section 3.3.

Proposition 2 *Suppose* $\phi_j = \phi$ *and* $\lambda > 0$. *Let* $\lambda'(\phi) \equiv \frac{\frac{\phi}{1-\phi}\mu(1-\bar{\beta})\bar{\beta}}{1+\frac{\phi}{1-\phi}\mu(1-\bar{\beta})\bar{\beta}}$ *and* $\lambda''(\phi) \equiv 1 - \frac{1-\phi}{\mu(1-\bar{\beta})}$.
(i) (Industry capture) If λ *is small and/or* ϕ *is large, i.e.,*

$$\lambda < \lambda'(\phi), \tag{12}$$

then $w_j = f_j$, $\Omega_j = \frac{\beta_j}{f_j} \frac{\bar{\beta}\mu}{\mu\bar{\beta}+\frac{\lambda}{1-\lambda}}$, *and* $\frac{\pi_j}{f_j} = 1 + \frac{\lambda}{1-\lambda} \frac{1}{\mu(1-\bar{\beta})\bar{\beta}}$.
(ii) (Constrained industry capture) If λ *and* ϕ *are intermediate, i.e.,*

$$\lambda'(\phi) \leq \lambda \leq \lambda''(\phi), \tag{13}$$

then $w_j = f_j$, $\Omega_j = \frac{\beta_j}{f_j} \frac{1-\phi}{1-\phi\bar{\beta}}$, *and* $\frac{\pi_j}{f_j} = \frac{1}{1-\phi}$.
(iii) (Oligarchy) If λ *is large and/or* ϕ *is small, i.e.,*

$$\lambda''(\phi) < \lambda \tag{14}$$

then $w_j > f_j$, $\Omega_j = \frac{\beta_j}{f_j} \frac{1-\lambda}{\frac{1}{\mu}+(1-\lambda)\bar{\beta}}$, *and* $\frac{\pi_j}{f_j} = \frac{1}{(1-\lambda)\mu(1-\bar{\beta})}$

Corollary 1 *As* λ *or* ϕ *increases, the number of firms decreases and profit increases (weakly) in every industry.*

Figure 1 illustrates Proposition 2 and the Corollary. Note that the equilibrium is symmetric across industries up to the scaling factor $\frac{\beta_j}{f_j}$, hence the discussion below applies to each industry. As λ rises, the leader attaches more importance to extracting income relative to raising welfare. Given the constraints, extracting more income is only possible if profits rise, which is accomplished by limiting the number of firms on the market. For low values of λ (case (i) of Proposition 2), profits remain relatively low, which limits clients' incentive to abscond. In this range, which we call "industry capture," the enforcement constraint is irrelevant and profit extraction can take place without giving rents to clients (i.e., $w_j = f_j$).[14]

At some point, however, the leader wishes to make profits so high that the enforcement constraint becomes binding (case (ii)). Profit extraction beyond this point becomes more costly as it requires providing rents to clients (i.e., $w_j > f_j$). For medium levels of λ, this extra cost is not worth it for the leader, and this halts the increase in profits (and the corresponding decline in competition and welfare). On Figure 1 this is indicated by the flat segment on each graph for medium values of λ. In this regime, which we call "constrained industry capture," the enforcement constraint limits the leader's incentive to extract income. This effect is larger the smaller is the leader's power ϕ. As shown on the Figure, a leader with smaller power ϕ will stop raising profits sooner (the threshold λ' shifts to the left), resulting in lower profit and hence more firms than under a more powerful leader. Thus, the model indicates a negative economic impact of stronger leaders in hybrid regimes. Because

[14] As the proof of Proposition 2 shows, in the optimal contract either the participation constraint or the enforcement constraint will bind. Intuitively, the PC $w_j \geq f_j$ already binds in the free entry equilibrium and hence the $\lambda = 0$ case (Proposition 1). With $\lambda > 0$, the leader has more incentive to lower w_j, so this constraint will still bind as λ rises above 0.

they are able to extract more surplus from their clients, such leaders have added incentives to increase profits. Because of this, welfare is unambiguously lower with a more powerful leader.[15]

For high λ, the leader values his private income so much that he is willing to incur the extra cost created by the enforcement constraint. In this regime, as λ increases, income extraction starts to rise again, with the associated increase in profits and decline in competition and welfare. However, unlike in the preceding cases of industry capture, the leader's clients receive some of the profits as rents - they become "oligarchs."

It is interesting to note that in general, the welfare effect of oligarchs depends on whether this regime emerges due to a high λ or a low ϕ. When oligarchs are created by a high value of extracted income λ, this is because the leader is willing to distort the economy *despite* turning clients into oligarchs, and welfare goes down. By contrast, when oligarchs are created by a reduction in ϕ (as is the case, e.g., for λ just below λ'' in the figure), this is because the leader is facing a limit in his ability to extract income. This lowers his incentive to distort, and welfare goes up. In this case, the presence of oligarchs acts as a moderating force on the leader's actions.

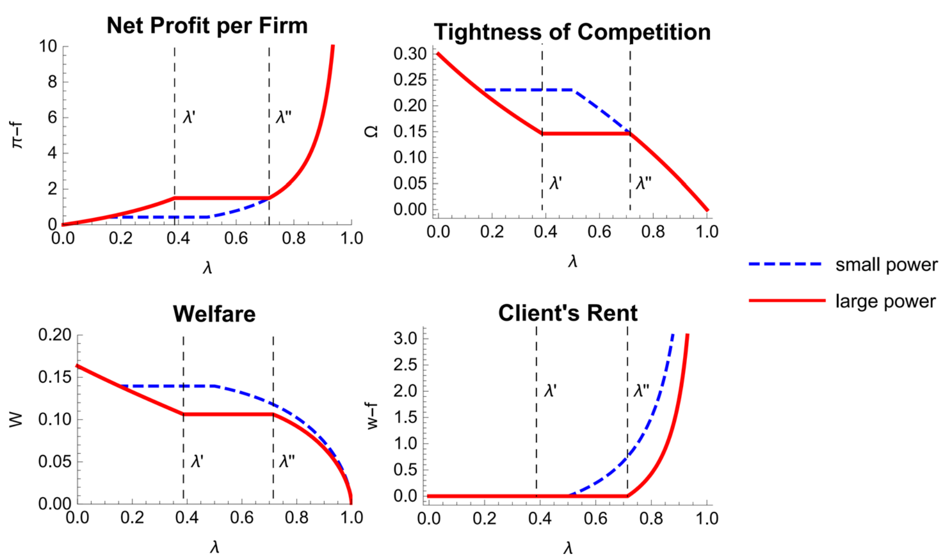

Figure 1: The effect of the value of extracted income, λ, and the leader's power, ϕ, on each firm's profit, industry competition, welfare, and each client's rent. $\lambda'(\phi)$ and $\lambda''(\phi)$ are the thresholds given in Proposition 2.

[15]Comparing Proposition 1 and 2 shows that $\lambda > 0$ results in a socially suboptimal number of firms Ω_j. Because a stronger leader reduces the number of firms, it follows that a stronger leader reduces welfare.

3.3 Spillovers across industries

Hybrid leaders may have clients in a few specific industries (e.g., natural resources), or in many industries throughout the economy. What is the impact of clientelism in one industry on other industries? This question is relevant for understanding the normative implications of clientelism, because the full welfare effect of the leader's power over his clients includes any spillover effects. The question is also relevant for thinking about the impact of policies that limit a leader's ability to extract income in some industries but not others.

To study these issues, we solve a version of the model where the leader has asymmetric power across industries. To maximize transparency, we assume $J = 2$ and drop the numeraire industry ($\beta_0 = 0$). The solution, which we present in detail in Appendix C, implies the following results.

Proposition 3 *Suppose that $\phi_1 = 0$. Then $\Omega_1 < \Omega_1^{FE}$ as long as $\phi_2 > 0$.*

Proposition 3 shows that a leader with asymmetric power across industries will limit entry even in industries over which he has no power. Clientelism in industry 2 (where the leader has power) spills over onto industry 1 (where he does not), and results in entry restrictions in *both* industries.

The reason for this is that a leader with no power over industry 1 can still raise income in the economy by raising profits in industry 1 through entry restrictions. Higher income means higher profits in *all* industries - including industries where $\phi_j > 0$, and whose income the leader is therefore able to extract. The idea is reminiscent of what d'Aspremont et al. (1996) have called the "Ford effect:" Henry Ford apparently observed that a (large) firm should take into account how increasing its price will, by raising profits, increase consumers' income, and therefore affect demand. In our case, instead of a firm setting prices, it is the leader setting entry regulations who optimally considers how general equilibrium effects can raise his profit.[16]

An immediate implication for institutional design is that eliminating the leader's power over a specific industry may by ineffective in limiting economic distortions. This is true even if the goal is to limit distortions in one industry only: due to the spillover effects, the free-entry outcome in *any* industry can only be achieved by eliminating the leader's power over *all* industries.

[16] Interestingly, the impact of these general equilibrium considerations differs from those typically seen in lobbying models. Lobbying tends to create asymmetries because a sector with an organized lobby wants different policies for itself than it does for other sectors. In Grossman and Helpman (1994), factor owners in lobbying sectors obtain trade protection for themselves, but promote competition in other sectors in order to lower prices on their personal consumption. By contrast, industry capture tends to create symmetries across industries, because the leader benefits from raising profits everywhere in the economy, including in industries that are not captured.

The next proposition asks whether more power implies more economic distortions. We pose this question in two ways: first, by comparing across industries, and second, by asking what happens if the leader's power over an industry increases, holding everything else constant.

Proposition 4 *1. Suppose that $\beta_1 = \beta_2$, $f_1 = f_2$, and $\phi_2 > \phi_1$. Then $\Omega_1 \geq \Omega_2$.*
2. Suppose that the leader's power in industry 1 (ϕ_1) rises. Then Ω_1 decreases and Ω_2 increases.

Part 1 of the proposition shows that, all else equal, distortions will be concentrated in industries where the leader has more power. This is intuitive, as it is more profitable for the leader to extract income when the clients' enforcement constraint is not binding. As distortions are increased, and clients' profits rise, this constraint will first become binding in the industry where the leader has less power. This limits the leader's incentive to distort in that industry.

According to part 2, an increase in the leader's power over industry 1, ϕ_1, reduces competition in industry 1 but increases it in industry 2. The own-industry effect is a generalization of the corresponding result from Corollary 1, and reflects the fact that more power raises the marginal utility of each additional dollar of profit for the leader. The cross-industry effect, however, goes in the opposite direction, which is due to an income effect. Because an increase in ϕ_1 leads to more profit extraction in industry 1, it increases the leader's income and reduces clients' income (and hence social welfare). This raises the marginal utility of increasing clients' income relative to the leader's private income, and this in turn incentivizes the leader to raise Ω_2.

Although in the symmetric case an increase in power was always detrimental to competition, with heterogeneous industries there are offsetting cross-industry effects. An increase in the leader's power over industry 1 can create enough income for the leader that he becomes more willing to increase competition in industry 2. In this sense, competition in some industries may benefit (in a second-best sense) from a leader with extensive powers over other industries in the economy.

The above analysis can also be used to shed light on the heterogeneity in clientelism across industries under a given leader. According to our model, this is driven by the value of extracted income λ and the distribution of the leader's power ϕ_j across industries. A high value of extracted income combined with similar power across industries will lead to a regime with oligarchs in several industries. Similar power across industries can arise, e.g., if the leader has a large network that allows for close monitoring of clients across the economy, or if few industries offer clients opportunities to abscond due to their dependence on local markets or natural resources. By contrast, when the leader's power is high in some industries but low in others, we expect to find oligarchs in the latter but not the former.[17]

4 Sanctions against hybrid regimes

The use of economic sanctions in foreign policy has increased dramatically since the end of the Cold War (Drezner, 2011), with the international response to Russia's invasion of Ukraine providing a salient recent example.[18] As noted by Morgan et al. (2023), "[o]ur theoretical and empirical understanding of sanctions has not kept up with these changes." (p5).

In most cases, sanctions are imposed on nondemocracies, with the aim of incentivizing a specific policy change and/or weakening the country's current leadership (Marinov, 2005). In order to understand sanctions' full impact, however, it is important to consider how the targeted leader might respond to them (Oechslin, 2014; De Bassa et al., 2021). One concern is whether, in equilibrium, sanctions could lead to excessive welfare losses for the population at large.

In this section, we use our framework to study how various sanctions impact (i) a leader's profit extraction, (ii) his incentive to create entry barriers, and (iii) the resulting change in social welfare relative to the leader's income. The latter is a relevant consideration to a sanctioner who wants to create policy changes, if the leader's incentive to change policy is tied to the income he loses as a result of the sanctions. It is also relevant to a sanctioner who wants to weaken the leader, if the leader's political fortunes depend on his income (e.g., if this income pays for the propaganda machine that ensures his electoral success). In these cases, from the sanctioner's perspective, reductions in the leader's income represent a benefit, while a decrease in social welfare is a cost.[19]

To demonstrate how our framework can shed light on the differential effects of various sanctions, we consider the following version of the leader's problem (11) in the symmetric case ($\phi_j = \phi$):

[17] Specifically, Proposition C.1 implies that both industries will have oligarchs if and only if $\lambda > max(\phi_1, \phi_2)$. By contrast, if $\lambda < \phi_2$, then industry 2 will not have oligarchs even if industry 1 does.

[18] Here we use the term "sanctions" very broadly to include trade restrictions, withholding international aid, freezing assets, etc.

[19] Although in our model lower social welfare also reduces the leader's utility, here we take the perspective of a sanctioner who views any reduction in social welfare as a cost. In reality, sanctions have other benefits and costs, including direct economic costs to the sanctioner, which we do not model.

$$\max_{\Omega,w} \lambda \ln \left(\sum_j \Omega_j (\pi_j - w_j) - B \right) + (1-\lambda) \ln \left(1 + \sum_j \Omega_j (w_j - f_j - C f_j) \right)$$
$$+ (1-\lambda) \sum_j \beta_j \mu \left(\ln \Omega_j - \ln \frac{c_j + A}{\mu - 1} \right) \quad (15)$$

with the participation and enforcement constraints (8)-(9) given by

$$w_j - C f_j \geq f_j \quad (16)$$
$$w_j - C f_j \geq \pi_j D (1 - \phi) \quad (17)$$

The parameters $A, B, C, D \geq 0$ represent various sanctions described below.

4.1 Example I: Broad sanctions increasing input costs

Common economic sanctions, like restrictions on a country's ability to purchase inputs or technology on the international market, lead to an increase in the costs of production ($A > 0$). On the one hand, this is costly for social welfare. On the other hand, in principle it is possible that the increase in production costs will hurt the leader, particularly when he obtains private income from firm profits.

Our model highlights a simple fact: the extent to which economic sanctions translate into profits, and hence the leader's income, depends on the nature of competition in the economy. In particular, if firms are able to fully pass on cost increases to their consumers by raising prices, then neither profits nor the leader's income will be affected. This is exactly what happens under monopolistic competition.

Proposition 5 *Sanctions increasing the cost of production (an increase in A) have no impact on profit extraction ($\pi - w$) or entry restrictions Ω. They lower social welfare W while leaving the leader's income Y_L unchanged.*

According to the proposition, in this model economic sanctions that raise production costs must have some other justification than a desire to disrupt the leader's profit extraction. More generally, the ineffectiveness of broad economic sanctions is consistent with the historical trend where broad sanctions are increasingly replaced by so-called "smart" sanctions that target specific actors and activities (Drezner, 2011).

4.2 Example II: Smart sanctions targeting the leader

Another form of sanctions aims to directly reduce the leader's income, for example, by freezing his foreign assets. We model this by increasing B in problem (15). To focus on the reduction of the leader's income, rather than the reduction of total income in the economy, we assume that the sanctioner spends B in the economy in the same way that the leader would have.[20]

Proposition 6 *Consider an oligarchy with no sanctions initially. A marginal increase in B lowers both the leader's income Y_L and social welfare W and leads to restricted competition. If $\lambda > \frac{1}{2}\left(1 + \frac{1}{\mu\beta}\right)$, then $\frac{\partial \ln W}{\partial B} / \frac{\partial \ln Y_L}{\partial B} > 1$.*

Naturally, B directly lowers the leader's income. However, this also raises its marginal utility, and this can give the leader an incentive to offset B by restricting competition and raising profits. This in turn lowers welfare. As the proposition shows, if λ is sufficiently large, so that the leader is motivated more by extracted income than social welfare, his effort to shield his interest are sufficiently strong that social welfare drops more than extracted income.

In this model, smart sanctions that target the leader are more effective at reducing the leader's income than broad economic sanctions in the sense of Proposition 5. However, directly targeting the leader's income is no "silver bullet" and may not avoid welfare losses once the leader optimally responds to the sanctions.

This simple idea is related to discussions of leaders' ability to directly offset sanctions - for example, by transferring resources to strategic firms hurt by the sanctions (Ahn and Ludema, 2020). Our analysis highlights that even a leader who cannot directly offset sanctions may be able to do so indirectly, by adjusting the economic policies that ultimately govern income extraction.

4.3 Example III: Smart sanctions targeting oligarchs

We contrast sanctions on the leader with two ways to target clients' income. First, the sanctioner could introduce a wedge C between the income the leader pays to the client and what the client can spend. This might be done by seizing a portion of the client's wealth held in foreign banks (a common form of sanctions, e.g., on Russian oligarchs). Note that in our formulation (15-17) this sanction is expected: C enters the equilibrium decision of both leader and clients and has general equilibrium consequences.

[20] In the main text, we highlight the results corresponding to an oligarchy, the most relevant parameter range for sanctions targeting the leader and his oligarchs. However, for completeness, Lemmas D.7-D.9 in the Appendix give a complete characterization for all λ.

Alternatively, sanctions may treat oligarchs who defect by absconding with their profits differently from those who stay with the leader. These differential sanctions are captured by the parameter D in (17). Namely, $D < 1$ if an oligarch absconding with his profits expects that some of it will be seized by foreign powers, while $D > 1$ captures a situation when a defecting oligarch enjoys an additional reward.

Proposition 7 *Consider an oligarchy with no sanctions initially.*

1. *A marginal increase in C lowers both the leader's income Y_L and social welfare W and leads to restricted competition. If $\lambda > \frac{1}{2}$, then $\frac{\partial \ln W}{\partial C} / \frac{\partial \ln Y_L}{\partial C} > 1$.*

2. *A reduction in D increases the leader's income Y_L and lowers social welfare: $\frac{\partial \ln W}{\partial D} > 0$, $\frac{\partial \ln Y_L}{\partial D} < 0$.*

If C increases, so that oligarchs expect part of their income-flows to be seized, they require larger transfers w from the leader to dissuade them from absconding. The leader's reaction is therefore similar to sanctions that target him directly, as in example II. To compensate for the lost income, the leader increases profits by restricting competition further. This in turn lowers welfare, and for sufficiently high λ this effect can be larger than the decline in the leader's income.

It is interesting to contrast these results with the effect of D, capturing the differential effect of sanctions on the (expected) income of defecting oligarchs. A reduction in D, i.e., a "tax" on defecting oligarchs' income, unambiguously decreases social welfare and increases the leader's income. Intuitively, if clients expect to be able to abscond with less, their terms of contracting with the leader worsens. Thus, the effect of a smaller D is akin to an increase in the leader's power ϕ. As we saw in section 3.2, a more powerful leader enjoys an improved trade-off between extracting funds and the corresponding welfare reduction. As a result, he extracts more even as this reduces welfare.

One lesson from this discussion is that replacing broad economic sanctions with smart sanctions that target the leader or his clients may not be sufficient to avoid negative welfare consequences. In our model, these sanctions are still too broad in that they are not focused on the *mechanism* through which the leader's income is generated. As illustrated by the example of sanctions on defecting oligarchs, sanctions that ignore the mechanism may inadvertently reinforce it, and lead to negative welfare consequences. By the same token, sanctions that disrupt the income extraction mechanism may be more successful. For example, the mirror image of the above argument is that ensuring higher incomes for defecting oligarchs (through an increase in D) would lead to less income extraction and higher welfare.[21]

[21] Rewarding defecting oligarchs is not without precedent: in July 2023, the UK removed Russian tycoon

In section 5, we will study other sanctions designed to disrupt income extraction in the context of public procurement.

5 Public procurement and sanctions affecting external transfers

Our analysis so far has focused on the economic role of the leader through the regulation of entry in markets for private goods. In practice, another important tool for leaders is the public procurement process, and hybrid leaders are notorious for obtaining private gains through this channel (see, e.g., Szűcs (2023)). How does public procurement interact with clientelism in hybrid regimes?

The procurement process also presents a major dilemma to potential sanctioners, because external funding can account for a significant portion of government expenditures. A prime example of this is Hungary, which relies on transfers from the European Union worth billions of Euros each year. Could withholding some of these funds be effective at weakening hybrid regimes, or are there other sanctions better suited for this?

To investigate these important questions, we modify our setup to include public procurement financed in part from domestic taxes and in part from external transfers. We explain the role of procurement in the hybrid leader's toolkit, then study the impact of sanctions in this context.

5.1 Income extraction and public procurement

Suppose that firms in the last industry, J, do not produce for the private market - instead, they produce (varieties of) public goods purchased by the government and consumed by the consumers. For example, industry J could be the road construction industry, with firms specializing in different types of roads, or roads in different geographic areas. To model the leader's extensive powers in dealing with firms in the public goods industry, we assume that the leader makes a take-it-or-leave it offer for the markup m to be paid over the cost c_J.[22]

The leader finances public procurement using a lump sum tax T and external funds Δ, and we assume that both of these are earmarked for the provision of public goods. Leaders

Oleg Tinkov from its list of sanctioned individuals after he spoke out against the invasion of Ukraine (`https://www.ft.com/content/fe6ab027-fb19-4593-9ef1-bb751aeeb14b`)

[22] As we show below, a welfare maximizing leader would set the same markup $m = \mu$ as the equilibrium of the baseline economy with no public goods (see (2)). Thus if $m > \mu$, then the leader chooses to overpay for these goods.

in hybrid regimes face constraints: they cannot simply pocket the taxes that the government collects or the transfers that international organizations provide.

By choosing m, Ω_J and T, the leader effectively decides on the share of income that society will allocate to the public good. Assuming that the leader does not tax his own income, denote this share with $\tau \equiv \frac{T}{Y-Y_L}$. Then the government's budget constraint can be written as

$$\tau\left(Y-\sum_j \Omega_j(\pi_j-w_j)\right)+\Delta = mc_J q_J \Omega_J, \tag{18}$$

where the left-hand-side is total revenue, and the right-hand-side is total spending on the products of industry J.

Just as in the baseline model, the chosen markup and quantity have to be such that firms are willing to participate in procurement, that is, the participation constraint (8) and the enforcement constraint (9) are satisfied for industry J.

We provide a detailed formulation of the general problem and a full characterization for a case when industries with private and public goods coexist in Appendix E.2. For our purposes, here it is sufficient to focus on the $J = 1$ case, that is, when the public good is produced by the single increasing-return-to-scale industry. The following Proposition describes the main properties of the equilibrium in this variant of our economy

Proposition 8 *Let* $J = 1$.

1. *The equilibrium features oligarchs in the sense of Proposition 2 if and only if* $\lambda > \lambda^{PP}(\phi)$ *(where* $\lambda^{PP}(\phi) > \lambda'(\phi)$*); otherwise, it features unconstrained industry capture.*

2. *For any* $\lambda > 0$ *and* $\Delta \geq 0$*, as the value of extracted income* λ *increases, the leader increasingly overprices public procurement (*$m > \mu$ *and* $\frac{\partial m}{\partial \lambda} > 0$*), and overspends on the public good (*$\frac{\partial \tau}{\partial \lambda} > 0$*) while providing less of it (*$\frac{\partial Q_J}{\partial \lambda} < 0$*). In addition, market concentration increases (*$\frac{\partial \Omega_J}{\partial \lambda} < 0$*).*

The equilibrium with procurement is illustrated on Figure 2, which also shows the corresponding no-procurement baseline for comparison. As before, the leader increases profits by restricting competition and extracts the resulting income, providing rents to oligarchs for λ high enough. In addition, the leader has an incentive to increase the cost of public goods by overpricing government procurement. By purchasing public goods at inflated prices and then extracting the elevated profits from his clients, the leader can effectively transform tax revenues (and external funds) into his private income. Even though taxes are earmarked for public goods, the combination of public procurement and clientelism allows the leader

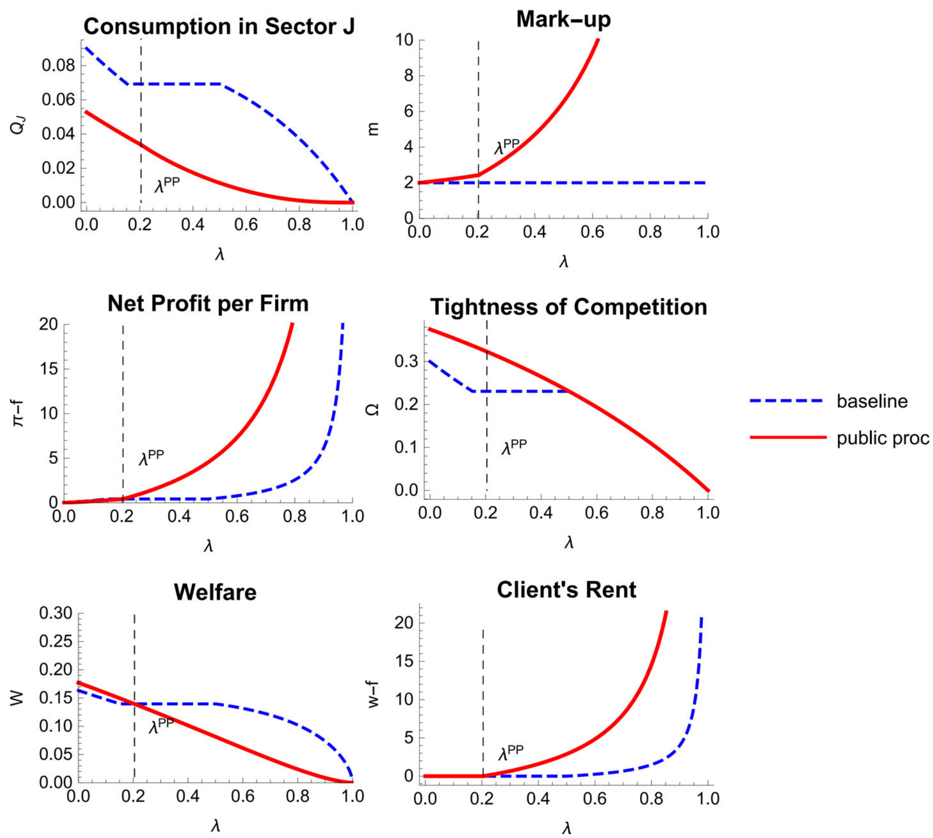

Figure 2: Clientelism with and without public procurement

to extract some of the surplus that is created. In this sense, the importance of public procurement to the leader derives precisely from the constraints he faces, namely his inability to divert tax revenue directly.[23]

5.2 Sanctions and procurement: Withholding external funds or improving the oversight of their allocation

Consider two potential interventions by an actor, such as an international organization, that provides external funds to a hybrid regime. First, external funds may be withheld. Second, the actor may try to improve oversight of the public procurement process where these funds are used in order to limit overpricing. The latter may be achieved through what is known as "conditionality" (Stokke, 2013): sanctions that are explicitly conditioned on specific policy

[23]As shown in the Figure, for low λ welfare with public procurement can be larger than in the baseline. This is because public procurement allows even a welfare-maximizing leader to alleviate the deadweight loss inherent in monopolistic competition. Thus, the welfare-maximizing solution is different with and without procurement (see Appendix E.2 for details).

changes. For example, as of 2022 the EU had suspended more than €13bn of funding to Hungary over concerns of democratic weaknesses and corruption. Improving the oversight of the procurement process and stamping out corruption in the allocation of these funds was one of the EU's main requirements to resume their flow.[24]

We contrast the effects of these two types of sanctions by comparing the equilibrium effect of reducing external funds, Δ, and of imposing an additional constraint $m \leq \bar{m}$ in the leader's problem.

Proposition 9 *1. A reduction in external funds, Δ implies smaller market concentration, $\frac{\partial \Omega_J}{\partial \Delta} > 0$, and less public good provision, $\frac{\partial Q_J}{\partial \Delta} > 0$ leading to both a reduction in welfare and the leader's income. For any λ and Δ, the relative effect is given by*

$$\frac{\partial \ln W}{\partial \Delta} / \frac{\partial \ln Y_L}{\partial \Delta} = \mu(1 - \hat{\beta}_0) + \hat{\beta}_0 > 1.$$

2. A stricter limit $\bar{m}$ on overpricing implies (weakly) smaller market concentration, $\frac{\partial \Omega_J}{\partial \bar{m}} \geq 0$, but more public good provision, $\frac{\partial Q_J}{\partial \bar{m}} < 0$, leading to a reduction in the leader's income but an increase in welfare:

$$\frac{\partial \ln Y_L}{\partial \bar{m}} > 0 > \frac{\partial \ln W}{\partial \bar{m}}.$$

The first part of the Proposition shows that withholding external funds has an unambiguously larger negative effect on welfare than on the leader's income. By contrast, the second part of the proposition shows that tighter limits on overpricing increase welfare while decreasing the leader's income. Thus, withholding external funds earmarked for public procurement and limiting overpricing in public procurement can have starkly different consequences.

The first sanction leads to a loss in available funds in the economy, which directly lowers both welfare and the leader's income. But because the leader controls tax revenues, markups, as well as entry, he has considerable flexibility in mitigating his losses by distributing them across the economy. In equilibrium, the leader responds to the decrease in his income by further restricting competition and reducing the quantity of the public good, which reduces welfare beyond the sanction's direct impact. The second intervention, instead of reducing funds in the economy, directly disrupts the mechanism the leader uses for income extraction, which raises welfare. Although the leader can undo this somewhat by limiting competition and increasing profits, he will extract less from the economy, and this results in *higher* welfare.

[24] https://www.theguardian.com/world/2022/nov/30/Brussels-seeking-to-freeze-13bn-of-eu-funds-to-hungary-over-corruption-fears

These results echo the findings in Section 4 showing that, unless interventions directly target the mechanism of income extraction, actions taken by the leader in order to protect his income can easily have undesirable welfare consequences.

6 Extensions

6.1 Innovation in hybrid regimes

In this section, we present a variant of our model to study the implications of hybrid regimes for innovation.

We model innovation intensity following the static version of Atkeson and Burstein (2010) introduced in Melitz and Redding (2014). Each entrepreneur, after entry, but before production, can choose effort $a \in [\underline{a}, 1]$ to increase productivity (innovate) for a convex cost $K(a)$. With probability a innovation is successful and the variable labor cost of production decreases to $\frac{c_j}{\hat{\kappa}}$, where $\hat{\kappa} > 1$. With probability $1-a$, the innovation is unsuccessful and the firm produces with variable cost c_j as in the baseline model. As the success of innovation is independent across firms, this extension results in ex-post heterogeneity across firms. For each variety ω a fraction a will be produced by high-productivity firms.

High-productivity firms charge lower prices, and sell larger quantities, earning higher profits (see Appendix F.1 for details). In particular, (5) is replaced by

$$\bar{\pi}_j = \frac{\beta_j}{\Omega_j} Y \frac{\kappa}{a\kappa + (1-a)} - K(a) \tag{19}$$

$$\underline{\pi}_j = \frac{\beta_j}{\Omega_j} Y \frac{1}{a\kappa + (1-a)} - K(a) \tag{20}$$

for high and low productivity firms, respectively, where $\kappa \equiv \hat{\kappa}^{\frac{1}{\mu-1}}$.

Crucially, we think of effort as unobservable to the leader. While the leader can offer different payments $\bar{w}_j$, $\underline{w}_j$ for entrepreneurs producing high and low profits, respectively, he cannot condition payment on effort. This introduces a layer of moral hazard into the leader-client contracting problem. By standard arguments, clients will choose effort consistent with the incentive constraint

$$\bar{w}_j - \underline{w}_j = K'(a). \tag{21}$$

Because of ex-post heterogeneity, high and low productivity firms also face different enforcement constraints. The enforcement constraint (9) is replaced by

$$\bar{w}_j \geq (1-\phi_j)\bar{\pi}_j \tag{22}$$

$$\underline{w}_j \geq (1-\phi_j)\underline{\pi}_j \tag{23}$$

Let $\tilde{\pi}_j = a\bar{\pi}_j + (1-a)\,\underline{\pi}_j$, and $\tilde{w}_j = a\bar{w}_j + (1-a)\underline{w}_j$ denote average profit and average compensation, respectively. It is easy to show that the participation constraint (8) changes only to the extent that (π_j, w_j) is replaced by $(\tilde{\pi}_j, \tilde{w}_j)$ and the leader's problem (11) becomes

$$\max_{\Omega,w} \lambda \ln\left(\sum_j \Omega_j(\tilde{\pi}_j - \tilde{w}_j)\right) + (1-\lambda)\left[\ln\left(1+\sum_j \Omega_j(\tilde{w}_j - f_j)\right) + \sum_j \beta_j \mu\left(\ln\Omega_j - \frac{\ln c_j}{\mu-1}\right)\right]$$
$$+ (1-\lambda)\sum_{j>0}\beta_j\mu\ln\left(\kappa a + (1-a)\right).$$

subject to (8) and (19)-(23).

As a benchmark we show in Appendix F.1 that just as in our baseline model, a welfare-maximizing leader's choices are exactly the $(a, \Omega, \bar{w}_j, \underline{w}_j)$ implied by free-entry. That is, the moral hazard problem does not affect the economy if the leader maximizes welfare. This is the case because a leader not interested in profit extraction can simply promise the full profit to the client, which aligns incentives perfectly.

To illustrate how this picture changes for the case of $\lambda > 0$, we solve the problem numerically for $J = 1$. We set the parameters such that the $\lambda = 0$ case is the same as in our baseline model illustrated in Figure 1.[25] Figure 3 presents the results.

Just as in the baseline case, there are regions of industry capture, constrained industry capture and oligarchy depending on which constraints bind. For instance, in an oligarchy both enforcement constraints (22)-(23) bind: clients earn rents as they have to be compensated for not absconding. However, as more productive clients could abscond with more, their compensation has to be higher.

We find that entrepreneurs' effort a_1 (and hence R&D spending $K(a_1)$ as well as productivity of the average firm, $\frac{\kappa a_1/+(1-a_1)}{c_1}$) is U-shaped in λ. Namely, effort is weakly decreasing in the regions outside oligarchy, and strictly increasing in oligarchy.

For low λ, the dominating effect is that incentivizing innovation is costly. As (23) starts to bind, reducing $\underline{w}$ is no longer possible, and innovation incentives can only come from an increase in $\bar{w}$. However, this would reduce extracted income, therefore as λ rises the leader

[25]For this, we set f_1, c_1 and κ in this extension so that $f_1 + K(a^{IFE})$ is equal to the fixed cost in the baseline, while $\frac{c_1}{\kappa a^{IFE}+(1-a^{IFE})}$ equals the variable cost in the baseline, where a^{IFE} denotes the leader's choice under free entry.

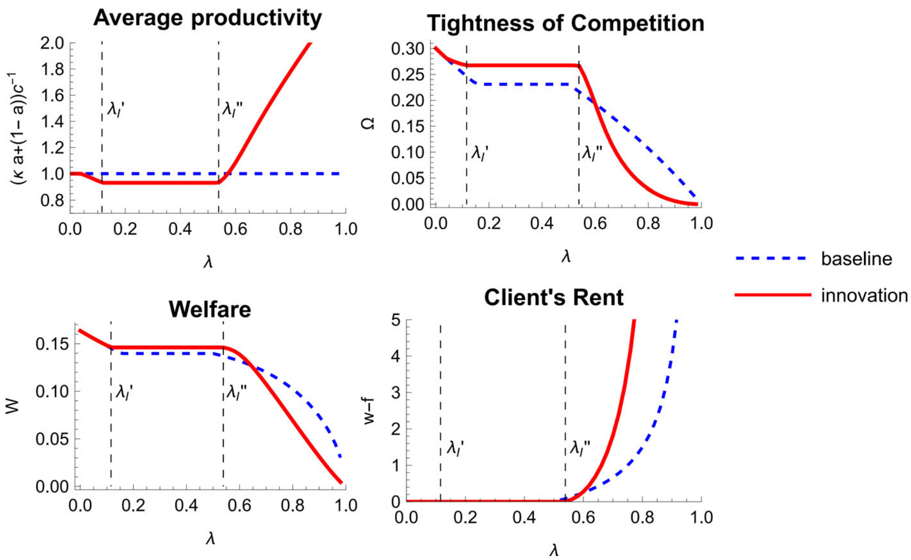

Figure 3: The economy with innovation (and moral hazard) and the baseline model. We choose parameters f_1, κ and c_1 for the economy with innovation such that first best productivity corresponds to productivity in baseline. Cost of effort is $K(a) = \frac{1}{1-a} - \frac{1}{1-\underline{a}}$. Parameters are $\kappa = 5, \underline{a} = 0.01, f_1 = 0.63, \beta_1 = 0.3, \mu = 2, c_1 = 2.1$.

opts for contracts that result in less innovation.

As λ increases into the oligarchy range, the incentives to innovate rise. The reason is that as the leader limits competition in order to increase profit levels, he also increases the dispersion of profits, $\bar{\pi} - \underline{\pi}$ (see (19)-(20)). Given binding enforcement constraints (22)-(23), a higher dispersion in profits translates into higher dispersion in rents, $\bar{w} - \underline{w}$, which in turn increases the incentives to innovate (equation (21)).

Thus, oligarchy may be associated with particularly high innovation. However, Figure 3 also shows that these high levels of innovation translate into low welfare. The reason is that the few firms who innovate do it at very high cost (as $K(\cdot)$ is convex), while producing few varieties and selling them at high prices. These patterns are consistent with evidence in Guriev and Rachinsky (2005) on the high TFP of firms owned by Russian oligarchs. According to our model, oligarchy is associated with suboptimal product variety, but high (even excessively high) productivity on existing products.

6.2 Favoring clients

In our model, the leader's objective has two components: his income and social welfare, where the latter weighs each consumer equally. In reality, the leader may attach extra weight to the welfare of his clients. For example, clients may provide the leader with political services, and happier clients could provide more services. Or, the leader may inherently care about entrepreneurs' well-being more than about workers' due to ideological reasons.

Intuitively, a higher weight on clients' welfare has two opposite effects. On the one hand, it creates an incentive for increasing the number of clients, which requires allowing more entry. On the other hand, when clients' welfare is increasing in profits, it creates an incentive to raise profits, which requires reducing entry. We now explore how these effects play out in our model and whether they have a significant impact on the solution.

Suppose each client in industry j gets a weight $\alpha_j \geq 1$ in the social welfare function used by the leader (while workers' weight remains 1). The new welfare expression then becomes

$$W\left(\Omega, w\right) = \hat{\beta}_0^{\hat{\beta}_0} \left(1 + \sum_j \Omega_j (\alpha_j w_j - f_j)\right) \prod_j \left(\frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j}\right)^{\hat{\beta}_j}.$$

Proposition 10 *1. When $\lambda = 0$, the solution is as given by Proposition 1.*
2. Suppose that $\lambda > 0$. Then in Proposition 2,
(i) the solution under Oligarchy remains unchanged;
(ii) under Industry capture,
- if $J = 1$, then α_1 raises the number of firms Ω_1 and shifts the threshold $\lambda'(\phi)$ up.
- if $J = 2$, then as α_1 rises holding α_2 constant, Ω_1 rises while Ω_2 falls.

Parts 1 and 2(i) of the proposition show that when clients' payment depends on profits (either in the welfare-maximizing solution or in the case of an oligarchy), the extra weights $\alpha_j > 1$ on clients' welfare have no impact on the solution. This is due to the general equilibrium nature of the model. To see why this is the case, consider a welfare-maximizing leader with $\lambda = 0$ (the intuition for oligarchy is analogous). Here the leader chooses $w_j = \pi_j$ and log weighted welfare can be written as

$$\ln\left(Y + \sum_j (\alpha_j - 1)\Omega_j \pi_j\right) + \sum_j \beta_j \mu \ln \Omega_j. \tag{24}$$

The first term is the log of weighted aggregate income, and shows that with increased weights $\alpha_j > 1$ on clients, the marginal utility of profits is higher *for given total income Y*. However, in general equilibrium total income and profits are determined jointly. It is easy to show that both Y and $\Omega_j \pi_j$ are proportional to workers' income, $1 - \sum_j \Omega_j f_j$. When plugged into (24), we see that α_j simply shifts welfare up or down, without affecting the marginal utility of Ω_j. Hence, the number of firms maximizing (24) is the same as in the model with $\alpha_j = 1$.

This implies that, in many cases $\alpha_j > 1$ has no impact on the model.

The only situation where the extra weight can make a difference is when clients receive no rents (Proposition 10 part 2(ii)), because here clients' payments are independent of profits ($w_j = f_j$). Now a higher weight for a particular industry gives the leader an incentive to have more clients in that industry, which is achieved by allowing more entry.

Interestingly, in this case there is a spillover effect on other industries: as the leader increases Ω_1, he lowers profits (and therefore his income) from industry 1. He can offset this loss by creating more profits in industry 2, by lowering the number of firms Ω_2. In this way, a larger weight on clients in industry 1 can lead to more entry in industry 1 and less entry in industry 2.

6.3 Rent-seeking and productive inefficiency

Our discussion so far has treated the share of profits $(1 - \phi)$ that a client could choose to abscond with as exogenous. In reality, clients may be able to exert effort and increase this share by investing in creative accounting methods, establishing offshore subsidiaries, or hiring managers loyal to them rather than to the leader. Indeed, practices such as tax evasion, fraud and corruption among client-entrepreneurs are well documented in hybrid regimes.

Suppose that an entrepreneur is able to reduce ϕ slightly. Doing this takes time and effort away from running the firm, and reduces productive efficiency. To model this, assume that if ϕ is lowered, the marginal cost of production c goes up.

Clearly, the client has no interest in lowering ϕ unless he receives rents. When λ is low enough that equilibrium clientelism involves no rents (cases (i) and (ii) of Proposition 2), lowering ϕ is never beneficial. On the other hand, when λ is so high that the leader is willing to provide rents in order to extract more income, these rents will give rise to socially costly rent-seeking. In an oligarchy, the client will *always* want to lower ϕ because, from his point of view, the only effect of this is a higher payment w. Because c goes up, this has a negative effect on welfare.[26] Thus, one potential downside of oligarchs in our model is that they have an incentive to engage in rent-seeking even at the expense of increased production costs and welfare losses.

[26]The net welfare effect depends on the relative size of the increase in the client's income (which raises welfare) and c (which lowers it).

7 Model and Facts

In this section we use our model to interpret a number of examples from hybrid regimes.

Oligarchs in Russia Russia offers two particularly famous, and historically consequential, examples. The first is a defining moment in the privatization of Soviet enterprises: President Yeltsin's controversial "loans-for-shares" program. This program sold off state companies at low prices to a group of oligarchs, who then provided resources to finance Yeltsin's 1996 reelection campaign (Shleifer and Treisman, 2005). The second example is from the early 2000s, when bargaining powers between President Putin and the oligarchs were markedly different. As described in the Dawisha quote in the Introduction, Putin extracted resources from oligarchs in return for letting them stay in business.

Based on our model, the Yeltsin episode reflects the start of an oligarchy in the sense of Proposition 2, as a leader with low power ϕ faces an increase in λ driven by the need for money to get reelected. This results in clients receiving rents. By contrast, the Putin episode reflects a leader whose increasing power ϕ allows him to reduce rents.

Our model predicts that both of these regimes will be associated with entry barriers and increased market concentration in order to deliver higher profits. As far as we know, this prediction has yet to be formally tested. It certainly seems plausible that a privatization program different from loans-for-shares may have resulted in more competition. Similarly, the Putin regime encouraged market concentration through mergers and the creation of conglomerates in several industries including aircraft design, shipbuilding, and defense (Aslund, 2019, p28).

Privatization in Latin America A common criticism of privatization programs around the world is that while they achieve the transfer of control and ownership of former state monopolies to the private sector, they do not necessarily increase competition, and monopolies remain. This was the case in the privatization programs of several Latin American countries in the 1990s (Manzetti, 1999). Our model provides an immediate explanation: market power (a low Ω) makes these companies more valuable to buyers, which is an important consideration to leaders if they hope to extract private income from these transactions.

For example, during President Menem's privatization program in Argentina "the symbiosis between political and economic power reached an unprecedented level as the largest conglomerates generously funded Menem's campaign re-election and their chief executive officers figured prominently on the list of special guests travelling abroad with the President to procure new business." (Manzetti, 1999, p140) These same conglomerates were the ones acquiring the privatized state companies. In some cases conglomerates expanded to new industries by acquiring state monopolies, while in sectors such petroleum or steel, they bought their previously state-owned competitors, thereby increasing their market power. (Manzetti, 1999, p135).

The License Raj in India In most cases, clientelistic contracts between the leader and private firms remain hidden, but in others they are part of a quasi-official system of favor exchange, justified by an ideology of promoting economic growth and equity. A salient example of this is India in the 1970s and 80s. One of Nehru's legacies was a development model based on central planning and government regulation. The stated goal was to facilitate the allocation of resources to high-priority industries to promote growth and accomplish various social objectives. But when the ruling Congress party faced shortages in party finances, the system transformed into what became known as the "License Raj," a system where Congress obtained funding from companies, while the latter "depended on the system to secure and maintain monopoly, protection, and guaranteed profitability." (Kochanek, 1987, p1284).

Over time, this system incentivized firms to evade taxes, and engage in black-market operations. On the one hand, this gave managers access to discretionary funds that could more easily be used for political payments; on the other hand, it all allowed reducing firms' assets that were visible to the state and that politicians could therefore make claims on (Root, 2006, Chapter 7). This illustrates the fundamental difficulty that hybrid leaders face in their efforts to extract resources: the possibility that cronies may shield some of these resources from them. Our model captures this through the enforcement constraint that clientelistic contracts are subject to.

Crony firms in North Africa Hybrid regimes in North Africa prior to the Arab Spring offer another set of examples consistent with our model. Systematic evidence exists on at least two countries, Tunisia and Morocco. Rijkers et al. (2017) document the economic success of firms connected to President Ben Ali, who ruled Tunisia between 1987-2011. Most industries in which these firms operated had two important characteristics: (i) they required government authorization for running a business, and (ii) they had restrictions on Foreign Direct Investment. In turn, these entry barriers allowed connected firms to generate abnormal profits.

Ruckteschler et al. (2022) study a trade liberalization episode between the EU and Morocco in 2000. They show that to offset the increased competition from foreign firms, Morocco introduced non-tariff measures (NTMs) such as input regulations, labeling requirements and shipping inspections. These protectionist measures were especially likely in industries with many "crony firms" – firms connected to politicians and to the royal family. Particularly interesting for our results is the fact that, although protection was more likely in industries with many cronies, *all* sectors where trade was liberalized experienced a subsequent rise in NTMs. This would be difficult to explain with either a welfare-maximizing government or a partial equilibrium model. However, it is consistent with our Proposition 3 which shows why, in general equilibrium, a leader with nonzero λ can benefit by restricting entry even in sectors in which he has no cronies.

Public procurement in Hungary In section 5, we showed how tax revenues and external funds can be channeled to private firms with the help of overpriced public procurement, and the resulting profits shared between the oligarchs and the leader. This process is well illustrated by the case of one of the highest net-worth Hungarians, Lorinc Meszaros.

Meszaros and Viktor Orban (the prime minister since 2010), became close in 1999 in Felcsut, a small village and Orban's birthplace. At the time Meszaros owned a small firm building gas pipelines in neighboring villages, and he sponsored the local football team where Orban played as a striker. Since 2010, Meszaros's net-worth has been increasing exponentially. It was estimated at $30M in 2013 when he first made it to the top 100 nationally. By 2021, it reached $1.5B, making him the second wealthiest Hungarian. By that time his activities had spread to other sectors, including public construction, energy, banking, agriculture, manufacturing, media, and hospitality.[27]

It is well documented that Meszaros's wealth accumulation was supported by the increasing share of public procurement contracts his firms won throughout the years. While this share was negligible in 2010, when Orban came to power, it grew to 5% of the value of *all* public procurement contracts by 2017, and to 17% by 2021. The bulk of these contracts were financed by EU funds.[28]

Investigative journalists have documented various ways in which, similarly to our model, Meszaros's profits are channeled back to support Orban. One common method is to use the procurement revenues to pay for subcontractors linked to the prime minister. One such subcontractor is Dolomit Ltd, a mining company owned by Gyozo Orban, the prime minister's father.[29] Another channel is through the media sector. In 2015 and 2016 Meszaros acquired several local and national newspapers as well as a national TV channel, then in 2017 gifted it all to a newly established foundation which has been running the government's

[27]See the Wikipedia article on Lőrinc Mészáros and citations therein.

[28]Estimates are from the non-profit Corruption Research Center Budapest, see `http://www.crcb.eu/?p=3400` and `http://www.crcb.eu/?p=3183`

[29]`https://www.direkt36.hu/en/igy-folyik-tovabb-a-kozpenz-meszaros-lorinctol-az-orban-csaladhoz/`

propaganda machine since.[30]

8 Conclusion

This paper seeks to shed light on the economics of hybrid regimes. To do this, we model industry capture by a leader who enters into clientelistic contracts with firms. These contracts create profits through entry restrictions, which are then divided between the leader and his clients as a function of the model's parameters. Two kinds of hybrid regimes emerge, one where clients obtain no rents, and one where they do and become oligarchs. We study the welfare implications of each regime in a number of scenarios, including heterogeneity in the leader's power, the presence of public procurement, and under different sanctions imposed by an outside actor such as an international organization.

When institutions are such that the leader has a high value from industry capture, market concentration and associated welfare losses will be particularly large. A leader with more power over clients also creates larger distortions, even as this is accompanied by a reduction in oligarchs' rents. In general equilibrium, entry restrictions by the leader will not be limited to industries from which he can extract income, but more power over specific industries can alleviate restrictions elsewhere in the economy. When the leader's ability to extract private income from tax revenues is constrained, he may be able to extract income indirectly, through clientelism in the context of public procurement. Finally, our model shows why even "smart sanctions" that directly target the leader's income may be costly for welfare, unless they can also target the mechanism responsible for creating this income, e.g., by limiting overpricing in procurement.

While most of the literature on non-democracies considers detailed models of the politics of these regimes, it treats the economy as a black box. Our paper takes the complementary approach of combining a detailed economic model with a reduced form treatment of politics. A natural next step in this research would be to combine these approaches, for example, by endogenizing the weights placed by the leader on private income and social welfare through explicit models of political competition or propaganda. Another useful extension would be to study different models of the economy, such as different market structures or allowing for international trade, in order to derive further economic implications of hybrid regimes. Finally, along the lines of our extension on innovation, a dynamic model with investment could shed light on the impact of hybrid leaders on economic development and growth.

[30] https://media1.hu/2019/06/05/meszaros-lorinc-mediaworks-talentis-vetelar-elajandekozas/

A Appendix: The benchmark economy

In this section, we derive (2)-(5), which take Ω_j as exogenously given. Utility maximization implies that consumer i's demand for product ω of industry $j > 0$ is

$$q_j(\omega, i) = \hat{\beta}_j Y(i) P_j^{\frac{1}{\mu-1}} p_j(\omega)^{\frac{\mu}{1-\mu}}, \tag{A.1}$$

where

$$P_j = \left[\int_0^{\Omega_j} p_j(\omega)^{\frac{1}{1-\mu}} \, d\omega \right]^{1-\mu}$$

is a price index. Demand for good 0 is $Q_0(i) = \hat{\beta}_0 Y(i)$. Aggregating across consumers, firms in industry $j > 0$ solve

$$\max_{p_j(\omega)} \hat{\beta}_j Y P_j^{\frac{1}{\mu-1}} \left(p_j(\omega) - c_j \right) p_j(\omega)^{\frac{\mu}{1-\mu}}$$

taking P_j as given. This yields the same price for each variety ω,

$$p_j(\omega) = \mu c_j, \tag{A.2}$$

and

$$P_j = \Omega_j^{1-\mu} \mu c_j. \tag{A.3}$$

Therefore, produced quantities are also the same within industry j,

$$q_j(\omega) = q_j = \hat{\beta}_j Y P_j^{\frac{1}{\mu-1}} \left(\mu_j c_j \right)^{\frac{\mu}{1-\mu}} = \frac{\beta_j Y}{\Omega_j(\mu-1)c_j}$$

where we used $\beta_j = \hat{\beta}_j \frac{\mu-1}{\mu}$ and (A.3).

The definition of Q_j gives

$$Q_j = \left[\int_0^{\Omega_j} q_j(\omega)^{\frac{1}{\mu}} \, d\omega \right]^{\mu} = \Omega_j^{\mu} q_j = \frac{\beta_j \Omega_j^{\mu-1}}{(\mu-1)c_j} Y$$

Profit, disregarding the fixed cost is

$$\pi_j = \hat{\beta}_j Y P_j^{\frac{1}{\mu-1}} p_j(\omega)^{\frac{\mu}{1-\mu}} \left(p_j(\omega) - c_j \right) = \frac{\beta_j}{\Omega_j} Y.$$

Using (A.1), (A.2) and (A.3), individual i's consumption is

$$Q_j(i) = \frac{\hat{\beta}_j \Omega_j^{\mu-1}}{\mu c_j} Y(i).$$

B Appendix: Proofs for Section 3.1 and 3.2

Proof of Proposition 1. Clearly, if $\lambda = 0$ then the leader does not extract any income, so $w_j = \pi_j$ and the enforcement constraint becomes irrelevant. The problem in (11) is

$$\max_{\Omega} \ln\left(1 + \sum_j \Omega_j\left(\pi_j - f_j\right)\right) + \sum_j \beta_j\left(\mu \ln \Omega_j - \frac{\mu}{\mu - 1}\ln c_j\right)$$

$s.t.$

$$\pi_k = \frac{\beta_k}{\Omega_k} Y \tag{A.4}$$

$$Y = 1 + \sum_j \Omega_j(\pi_j - f_j) \tag{A.5}$$

$$\pi_k \geq f_k \tag{A.6}$$

Using (A.4) and the fact that $\sum_{j>0} \beta_j = \frac{\mu-1}{\mu} - \beta_0$, (A.5) can be expressed as

$$Y = \mu \frac{1 - \sum_j \Omega_j f_j}{\mu\beta_0 + 1}. \tag{A.7}$$

Then the problem becomes

$$\max_{\Omega} \ln\left(1 - \sum_j \Omega_j f_j\right) + \sum_j \beta_j\left(\mu \ln \Omega_j - \frac{\mu}{\mu - 1}\ln c_j\right)$$

$$s.t. \frac{1 - \sum_j \Omega_j f_j}{\mu\beta_0 + 1}\frac{\mu\beta_k}{\Omega_k} \geq f_k$$

The derivative of the objective w.r.t. Ω_k is

$$\frac{-f_k}{1 - \sum \Omega_j f_j} + \frac{\mu\beta_k}{\Omega_k}. \tag{A.8}$$

If the constraint is slack for any k, then $\frac{-f_k}{1-\sum \Omega_j f_j} + \frac{\mu\beta_k}{\Omega_k} > \frac{-1}{\mu\beta_0+1}\frac{\mu\beta_k}{\Omega_k} + \frac{\mu\beta_k}{\Omega_k} > 0$, so this cannot be an optimum. If all constraints bind, then $Y = 1$ and hence $\pi_k = \frac{\beta_k}{\Omega_k}$, which in turn implies that (A.8) is equal to 0 so we have an optimal solution. ■

Proof of Proposition 2 We prove the proposition through a series of Lemmas.

Lemma B.1 *At least one of the constraints (8) or (9) must bind for every industry j.*

Proof. Substituting A.7 into A.4, define

$$\pi_j\left(\Omega_J\right) \equiv \frac{\beta_j}{\Omega_j}\frac{1}{1 - \bar{\beta}}\left(1 - \sum_{j=1}^{J} \Omega_j f_j\right) \tag{A.9}$$

Then, we can write the Lagrangian corresponding to problem (11) as

$$\max_{\Omega,w} \lambda \ln \sum_j \Omega_j(\pi_j(\Omega) - w_j) + (1-\lambda)\ln[1 + \sum_j \Omega_j(w_j - f_j)] + (1-\lambda)\sum_j \beta_j \mu \ln \Omega_j$$
$$- \sum_j \gamma^j_{PC}(w_j - f_j) - \sum_j \gamma^j_{EC}[w_j - (1-\phi_j)\pi_j(\Omega)], \quad \text{(A.10)}$$

where and $\gamma^j_{PC} \geq 0$ and $\gamma^j_{EC} \geq 0$ are the Lagrange multipliers corresponding to the PC and EC constraints of industry j, respectively.

The first-order conditions for w_k and Ω_k can be written as

$$\frac{-\lambda}{\sum_j \Omega_j(\pi_j(\Omega) - w_j)} + \frac{1-\lambda}{1 + \sum_j \Omega_j(w_j - f_j)} - \frac{\gamma^k_{EC} + \gamma^k_{PC}}{\Omega_k} = 0 \quad \text{(A.11)}$$

$$\lambda \frac{-\frac{f_k\bar{\beta}}{1-\bar{\beta}} - w_k}{\sum_j \Omega_j(\pi_j(\Omega) - w_j)} + (1-\lambda)\frac{w_k - f_k}{1 + \sum_j \Omega_j(w_j - f_j)} + (1-\lambda)\frac{\beta_k \mu}{\Omega_k} + \sum_j (1-\phi_j)\gamma^j_{EC}\frac{\partial \pi_j(\Omega)}{\partial \Omega_k} = 0 \quad \text{(A.12)}$$

Suppose both constraints were slack for some industry j'. Then $\gamma^{j'}_{PC} = \gamma^{j'}_{EC} = 0$, therefore

$$\frac{\lambda}{\sum_j \Omega_j(\pi_j(\Omega) - w_j)} = \frac{1-\lambda}{1 + \sum_j \Omega_j(w_j - f_j)} \quad \text{(A.13)}$$

from (A.11). But because (A.13) is independent of j', (A.11) implies that we must also have $\gamma^k_{PC} + \gamma^k_{EC} = 0$ for all $k \neq j'$. This is only possible if $\gamma^k_{PC} = \gamma^k_{EC} = 0$ for all k.

Using this observation together with (A.9) and (A.11), (A.12) can be rewritten as

$$\beta_k \mu \left(1 - \bar{\beta}\right)\left(1 + \sum_j \Omega_j(w_j - f_j)\right) = f_k \Omega_k \ \forall k. \quad \text{(A.14)}$$

Using (A.9), (A.13) can be written as

$$\frac{\lambda}{\frac{\bar{\beta}}{1-\bar{\beta}}(1 - \sum_j \Omega_j f_j) - \sum_j \Omega_j w_j} = \frac{1-\lambda}{(1 - \sum_j \Omega_j f_j) + \sum_j \Omega_j w_j}$$

implying

$$\sum_j \Omega_j w_j = \left((1-\lambda)\frac{\bar{\beta}}{1-\bar{\beta}} - \lambda\right)\left(1 - \sum_{j=1}^{J} \Omega_j f_j\right). \quad \text{(A.15)}$$

Substituting (A.15) into (A.14), we obtain

$$f_k \Omega_k = \beta_k \mu (1-\lambda)(1 - \sum_j \Omega_j f_j) \ \forall k. \quad \text{(A.16)}$$

implying

$$\bar{\beta}\mu\,(1-\lambda)\left(1-\sum_{j=1}^{J}\Omega_j f_j\right)=\sum_{j=1}^{J}\Omega_j f_j.$$

Our starting assumption was that the participation constraint (8) is slack for at least one industry. This would imply $\sum_j \Omega_j w_j > \sum_j \Omega_j f_j$. Using (A.15), this would mean

$$\frac{\bar{\beta}\,(1-\lambda)}{1-\bar{\beta}}-\lambda>\bar{\beta}\mu\,(1-\lambda)$$

which in turn would require

$$\frac{1}{1-\bar{\beta}}>\mu$$

or $\mu\beta_0<0$. This is not possible, hence at least one of the constraints must bind for every industry. ■

Lemma B.2 *Let $\phi_j=\phi$ for all j. If*

$$\lambda<\lambda'\,(\phi)\equiv\frac{\frac{\phi}{1-\phi}\mu(1-\bar{\beta})\bar{\beta}}{1+\frac{\phi}{1-\phi}\mu(1-\bar{\beta})\bar{\beta}}$$

the optimal solution is

$$\begin{aligned}w_j&=f_j\\ \Omega_j&=\frac{(1-\lambda)\,\bar{\beta}\mu}{\left(\lambda+(1-\lambda)\,\bar{\beta}\mu\right)}\frac{\beta_j}{f_j}\end{aligned}$$

implying

$$\frac{\pi_j}{f_j}=1+\frac{\lambda}{1-\lambda}\frac{1}{\mu(1-\bar{\beta})\bar{\beta}}.$$

Proof. We consider the problem where the EC's (9) are ignored for every j. We show that in this relaxed problem the solution in the statement is optimal. Then, we show that under the restriction on λ, all constraints (9) are slack. Hence, the proposed solution remains optimal in the original problem.

Note first that if all the EC's are slack, then by Lemma B.1, each PC has to bind, that is $w_j=f_j$. But in this case, (A.12) implies

$$(1-\lambda)\frac{\beta_k\mu}{\Omega_k f_k}=\frac{\lambda}{1-\bar{\beta}}\frac{1}{\sum_j\Omega_j(\pi_j(\Omega)-f_j)}$$

substituting in (A.9) and summing up implies

$$\frac{\frac{1-\lambda}{\lambda}\bar{\beta}^2\mu}{1+\frac{1-\lambda}{\lambda}\bar{\beta}\mu}=\sum_{j=1}^{J}\Omega_j f_j$$

hence

$$\frac{(1-\lambda)\,\bar{\beta}\mu}{\left(\lambda+(1-\lambda)\,\bar{\beta}\mu\right)}\frac{\beta_k}{f_k}=\Omega_k.$$

Therefore, by (A.9) profit per firm is

$$\frac{\pi_j}{f_j}=\frac{\lambda}{\left(1-\bar{\beta}\right)(1-\lambda)\,\bar{\beta}\mu}+1$$

Hence, as long as

$$\frac{\lambda}{\left(1-\bar{\beta}\right)(1-\lambda)\,\bar{\beta}\mu}+1<\frac{1}{1-\phi}$$

this solution satisfies all (9) constraints. Rearranging, we obtain the condition in the Lemma. ■

Lemma B.3 *Let $\phi_j=\phi$ for all j. If*

$$\lambda>\lambda''\left(\phi\right)\equiv 1-\frac{1-\phi}{\mu(1-\bar{\beta})}$$

the optimal solution is

$$\begin{aligned}\Omega_j&=\frac{\beta_j}{f_j}\frac{(1-\lambda)}{\frac{1}{\mu}+(1-\lambda)\bar{\beta}}\\ \frac{\pi_j}{f_j}&=\frac{1}{(1-\lambda)\mu(1-\bar{\beta})}\end{aligned}$$

Proof. Consider the problem where we ignore (8) for all j. By Lemma B.1, we know that in this case all EC's must bind, i.e.,

$$w_j=(1-\phi)\,\pi_j\left(\Omega\right).$$

The problem of the leader simplifies to

$$\max_{\Omega}\lambda\ln\phi\sum_j\Omega_j\pi_j(\Omega)+(1-\lambda)\ln[1+\sum_j\Omega_j((1-\phi)\,\pi_j\left(\Omega\right)-f_j)]+(1-\lambda)\sum_j\beta_j\mu\ln\Omega_j.$$

Substituting in (A.9) and omitting constants, this simplifies to

$$\max_{\Omega}\ln\left(1-\sum_{j=1}^{J}\Omega_jf_j\right)+(1-\lambda)\sum_j\beta_j\mu\ln\Omega_j$$

The FOC w.r.t. Ω_j is

$$(1-\lambda)\beta_j\mu\left(1-\sum_{j=1}^{J}\Omega_j f_j\right)=f_j\Omega_j$$

which gives

$$\frac{(1-\lambda)\bar{\beta}\mu}{1+(1-\lambda)\bar{\beta}\mu}=\sum_{j=1}^{J}f_j\Omega_j$$

Hence

$$\frac{\beta_j\mu}{f_j}\frac{(1-\lambda)}{1+(1-\lambda)\bar{\beta}\mu}=\Omega_j$$

and substituting back to (A.9)

$$\pi_j=\frac{f_j}{\mu\left(1-\bar{\beta}\right)(1-\lambda)}$$

The solution satisfies each PC (8) if

$$\frac{(1-\phi)}{\mu\left(1-\bar{\beta}\right)(1-\lambda)}>1$$

which gives the condition in the statement. ■

Lemma B.4 *Let $\phi_j=\phi$ for all j. If*

$$\lambda'(\phi)\leq\lambda\leq\lambda''(\phi)$$

then

$$w_j=f_j,\ \frac{\pi_j}{f_j}=\frac{1}{1-\phi},\ \Omega_j\equiv\frac{\beta_j}{f_j}\frac{1-\phi}{1-\phi\bar{\beta}}.$$

Proof. Note that for the proposed solution all ECs (9) and PCs (8) bind. Note also that substituting in the proposed solution

$$\sum_j\Omega_j\pi_j(\Omega)-\sum_j\Omega_j f_j=\frac{\bar{\beta}\phi}{1-\phi\bar{\beta}}$$

and (A.11) gives

$$1-\lambda\frac{1}{\bar{\beta}\phi}=\frac{\gamma_{EC}^k+\gamma_{PC}^k}{\Omega_k}. \tag{A.17}$$

Also as

$$\frac{\partial \pi_k(\Omega)}{\partial \Omega_k} = \frac{1}{1-\bar{\beta}} \frac{\beta_k}{\Omega_k} \left(-\frac{1}{\Omega_k} + \frac{1}{\Omega_k} \sum_{j=1}^{J} \Omega_j f_j - f_k \right) = -\frac{1}{1-\bar{\beta}} \frac{f_k^2}{\frac{1-\phi}{1-\phi\bar{\beta}}} \left(\frac{1-\phi\bar{\beta}}{\beta_k (1-\phi)} + 1 - \frac{\bar{\beta}}{\beta_k} \right)$$

$$\frac{\partial \pi_j(\Omega)}{\partial \Omega_k} = -\frac{1}{1-\bar{\beta}} \frac{\beta_j}{\Omega_j} f_k = -\frac{1}{1-\bar{\beta}} \frac{1-\phi\bar{\beta}}{1-\phi} f_j f_k$$

(A.12) implies

$$\beta_k \frac{-\lambda \frac{1}{\bar{\beta}\phi\left(1-\bar{\beta}\right)} + (1-\lambda)\frac{\mu}{1-\phi} - \frac{1}{1-\bar{\beta}} \sum_j \gamma_{EC}^j f_j}{\frac{1}{1-\bar{\beta}} \left(\frac{1-\phi\bar{\beta}}{1-\phi} - \bar{\beta} \right)} = \gamma_{EC}^k f_k. \tag{A.18}$$

Summing up by industry and solving for $\sum_{j>0} \gamma_{EC}^j f_j$ gives

$$\frac{\bar{\beta} \frac{-\lambda \frac{1}{\bar{\beta}\phi} + (1-\lambda)\frac{(1-\bar{\beta})\mu}{1-\phi}}{\frac{1-\phi\bar{\beta}}{1-\phi} - \bar{\beta}}}{\frac{\bar{\beta}\phi - 1}{\bar{\beta} - 1}} = \sum_k \gamma_{EC}^k f_k$$

and substituting back to (A.18) implies

$$\frac{\bar{\beta}\mu\phi (1-\lambda) \left(1-\bar{\beta}\right) - \lambda (1-\phi)}{\bar{\beta}\phi \left(1-\bar{\beta}\phi\right)} \frac{\beta_k}{f_k} = \gamma_{EC}^k.$$

Using (A.17) also gives

$$\frac{-\mu - \phi + \beta\mu + \lambda\mu - \beta\lambda\mu + 1}{1-\phi} = \frac{\gamma_{PC}^k}{\Omega_k}$$

These two expressions imply that $\gamma_{EC}^k \geq 0$ if $\lambda \geq \lambda'(\phi)$ while $\gamma_{PC}^k \geq 0$ if $\lambda \leq \lambda''(\phi)$. Hence under the condition of the Lemma, the proposed solution solves the Lagrangian (A.10). ■

Proof of Corollary 1. Based on Proposition 2, it is easy to verify that Ω_j is continuous in $(1-\phi)$. Raising $(1-\phi)$ moves the solution from Industry capture to Constrained industry capture to Oligarchy. Under Constrained industry capture, it increases the number of firms since $\frac{\partial \Omega_j}{\partial \phi} < 0$. In addition, *holding all else fixed*, Ω_j is smaller under Industry capture than under Oligarchy. Thus, a higher ϕ weakly reduces the number of firms.

As the conditions in the proposition make clear, as λ rises, we move from Industry capture to Constrained industry capture to Oligarchy. In addition, note that $\frac{\partial \Omega_j}{\partial \lambda} < 0$ under Industry capture and Oligarchy, while Ω_j is independent of λ under Constrained industry capture. Again, it is easy to verify that Ω_j is continuous in λ. Thus, raising λ reduces the number of firms. ■

C Appendix: The asymmetric case (proofs for Section 3.3)

The following Lemma, which characterizes the case where different constraints are slack in each industry, will be used extensively.

Lemma C.5 *Let $J = 2$ and assume that only EC1 and PC2 bind. Then $w_1 > f_1$, $w_2 = f_2$, $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and*

$$\Omega_2 = \frac{1}{f_2}\frac{(2A(\mu(1-A)-1)+1)(1-\lambda)\mu\beta_2 + \lambda + A(\mu(1-A)-1) - S}{2A\mu(1-A)(\lambda+\mu-\lambda\mu)} \tag{A.19}$$

where $A \equiv (1-\phi_1)\beta_1$ and

$$S \equiv \sqrt{\mu^2(\lambda-1)^2\beta_2^2 - 2\mu(1-\lambda)(2A\lambda - A - \lambda + A\mu(2\lambda-1)(A-1))\beta_2 + (A-\lambda-A\mu+A^2\mu)^2}$$

.

Proof of Lemma C.5. Assume that only EC1 and PC2 bind: $w_2 = f_2$ and $w_1 = (1-\phi)\pi_1 > f_1$. The latter implies

$$\Omega_1 f_1 < (1-\phi_1)\mu\beta_1(1-\sum_j \Omega_j f_j). \tag{A.20}$$

Define $\xi \equiv \frac{\Omega_2 f_2}{1-\Omega_1 f_1}$. We know that $A < \frac{\mu-1}{\mu} = \sum_j \beta_j$, and $\xi \in [0,1]$ because $1 \geq \sum_j \Omega_j f_j$.

The first-order conditions w.r.t. Ω_1 and Ω_2 can be written respectively as

$$\frac{-(\mu-1-\mu A)\lambda}{(1-\xi)(\mu-1-\mu A)-\xi}\frac{\Omega_1 f_1}{1-\Omega_1 f_1} + \frac{1-\lambda}{1+\mu A(1-\xi)}(-\mu A-1)\frac{\Omega_1 f_1}{1-\Omega_1 f_1} + (1-\lambda)\mu\beta_1 = 0 \tag{A.21}$$

$$\frac{-\xi\mu\lambda(1-A)}{(1-\xi)(\mu-1-\mu A)-\xi} - \frac{1-\lambda}{1+\mu A(1-\xi)}\mu A\xi + (1-\lambda)\mu\beta_2 = 0. \tag{A.22}$$

Expression (A.22) yields

$$F(\xi) \equiv (1-\xi)^2\mu^2(1-A)A(1-\lambda)\beta_2 + (1-\xi)\mu[(1-2A)(1-\lambda)\beta_2 - \xi A(1-A)] - \xi(\lambda-A) - (1-\lambda)\beta_2 = 0. \tag{A.23}$$

This is a quadratic equation in ξ. Solving, it can be verified that only the lower root satisfies $\pi_2 \geq f_2$. This is

$$\xi(\phi_1) = \frac{(2A(\mu(1-A)-1)+1)(1-\lambda)\mu\beta_2 + \lambda + A(\mu(1-A)-1) - S}{2A\mu(\mu\beta_2(1-\lambda)+1)(1-A)}, \tag{A.24}$$

where S is defined in the statement of the proposition. Substituting (A.24) into (A.21) yields

$$\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}. \tag{A.25}$$

Substituting this into (A.24) and solving for Ω_2 yields (A.19). ■

Lemma C.6 *For $\xi(\phi_1)$ given by (A.24), $\frac{\partial \xi}{\partial A} < 0$.*

Proof. Because the denominator of (A.24) is positive, we have that $\frac{\partial \xi}{\partial A}$ is proportional to

$$2\left(\mu(1-2A)-1\right)(1-\lambda)\mu\beta_2 + \mu(1-2A) - 1 - \frac{\partial S}{\partial A} - 2\mu\left(\mu\beta_2(1-\lambda)+1\right)(1-2A)\,\xi.$$

Taking the derivative of S and using (A.24), algebra shows that this expression is always negative. ■

The following Proposition describes situations where the leader has little power in one of the industries (industry 1).

Proposition C.1 *Suppose that ϕ_1 is small enough that $PC1$ is slack. Then $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and there exists $\widetilde{\phi}_2 \in (\lambda, 1)$ such that*
(i) if $\phi_2 < \lambda$, then $w_1 > f_1$, $w_2 > f_2$, and $\Omega_2 = \frac{\beta_2}{f_2}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$.
(ii) if $\widetilde{\phi}_2 < \phi_2$, then $w_1 > f_1$, $w_2 = f_2$, and Ω_2 is given by (A.19)
(iii) if $\lambda \le \phi_2 \le \widetilde{\phi}_2$, then $w_1 > f_1$, $w_2 = f_2$, and $\Omega_2 = \frac{\beta_2}{f_2}\frac{\mu(\lambda+(1-\lambda)(1+\mu\beta_2))}{(\frac{1}{1-\phi_2}+\mu\beta_2)(\lambda+\mu-\lambda\mu)}$.

Proof of Proposition C.1. By assumption PC1 is slack (which will be the case, e.g., for $\phi_1 = 0$), therefore EC1 binds. There are only 3 cases to consider for $j = 2$'s constraints: only PC2 binds, only EC2 binds, both PC2 and EC2 bind.

Step 1. Assume that only PC2 binds: $w_2 = f_2 > (1-\phi_2)\pi_2$, which implies

$$\Omega_2 f_2 > (1-\phi_2)\mu\beta_2(1-\sum_j \Omega_j f_j). \tag{A.26}$$

From Lemma C.5, we have $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and Ω_2 is given by (A.19). Using the notation $\xi(\phi_1)$ from (A.24), (A.26) can be rewritten as

$$\xi(\phi_1) > \frac{\mu(1-\phi_2)\beta_2}{1+(1-\phi_2)\beta_2\mu}. \tag{A.27}$$

Note that (A.27) holds for $\phi_2 = 1$, and the right-hand side is decreasing in ϕ_2. Using (A.24), it can also be shown that (A.27) fails for $\phi_2 = \lambda$. It follows that there must be some $\widetilde{\phi}_2 \in (\lambda, 1)$ such that (A.27) holds if and only if $\phi_2 > \widetilde{\phi}_2$.

Step 2. Assume that only EC2 binds: $w_2 = (1-\phi_2)\pi_2 > f_2$. Then the first order condition w.r.t. Ω_k is

$$\frac{-f_k}{1-\sum_j \Omega_j f_j} + (1-\lambda)\frac{\beta_k \mu}{\Omega_k} = 0,$$

which can be solved to obtain $\Omega_k = \frac{\beta_k}{f_k}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$. The condition $(1-\phi_2)\pi_2 > f_2$ yields $\lambda > \phi_2$.

Step 3. As shown in Step 1, condition (A.27) fails for $\phi_2 \leq \lambda < \widetilde{\phi}_2$. Hence if a solution where only PC2 binds is feasible, then a solution where only EC2 binds is not, and vice versa. Since the only other possibility (when both PC2 and EC2 binds) imposes an extra constraint relative to either of these, that solution cannot be optimal when one of these less constrained solutions is feasible.

Step 4. The only remaining case is when $\lambda < \phi_2 < \widetilde{\phi}_2$. Then it must be that both PC2 and EC2 bind: $\Omega_2 f_2 = \mu A(1 - \sum_j \Omega_j f_j)$, from which

$$\Omega_2 f_2 = \frac{\mu(1-\phi_2)\beta_2(1-\Omega_1 f_1)}{1+\mu(1-\phi_2)\beta_2}.$$

Substituting into the objective, and solving, we obtain $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$ and $\Omega_2 = \frac{\beta_2}{f_2}\frac{\mu(\lambda+(1-\lambda)(1+\mu\beta_2))}{(\frac{1}{1-\phi_2}+\mu\beta_2)(\lambda+\mu-\lambda\mu)}$. ■

The following Proposition turns to cases where the leader's power in one of the industries (industry 2) is large.

Proposition C.2 *Suppose that ϕ_2 is large enough that EC2 is slack. There exists $\widetilde{\phi}_1 \in (0, \frac{\mu\lambda}{\lambda+\mu-1})$ such that*

(i) if $\frac{\mu\lambda}{\lambda+\mu-1} < \phi_1$, then $w_1 = f_1$, $w_2 = f_2$, and $\Omega_j = \frac{\beta_j}{f_j}\frac{(\mu-1)(1-\lambda)}{\lambda+(\mu-1)(1-\lambda)}$, $j = 1, 2$.

(ii) if $\phi_1 < \widetilde{\phi}_1$, then $w_1 > f_1$, $w_2 = f_2$, $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and Ω_2 is given by (A.19)

(iii) if $\widetilde{\phi}_1 \leq \phi_1 \leq \frac{\mu\lambda}{\lambda+\mu-1}$, then $w_1 = f_1$, $w_2 = f_2$, Ω_2 is the lower root of

$$-\frac{\lambda}{1-\lambda}\mu f_2\Omega_2(1-f_2\Omega_2) + [\mu(1-f_2\Omega_2) - 1 - \mu A][\mu\beta_2 - (\mu-1)f_2\Omega_2] = 0, \tag{A.28}$$

and $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\phi_1)(1-\Omega_2 f_2)}{1+(1-\phi_1)\mu\beta_1}$.

Proof of Proposition C.2. By assumption EC2 is slack (which will be the case, e.g., for $\phi_2 = 1$), therefore PC2 binds. There are only 3 cases to consider for $j = 1$'s constraints: only PC1 binds, only EC1 binds, both PC1 and EC1 bind.

Step 1. Assume that only PC1 binds, $w_1 = f_1 > (1-\phi_1)\pi_1$, which implies

$$\Omega_1 f_1 > (1-\phi_1)\mu\beta_1(1-\sum_j \Omega_j f_j). \tag{A.29}$$

Then the first order condition w.r.t. Ω_k is

$$\lambda \frac{-f_k \mu}{\sum_j \Omega_j (\pi_j(\Omega) - f_j)} + (1-\lambda)\frac{\beta_k \mu}{\Omega_k} =$$
$$\lambda \frac{-f_k \mu}{\mu(1-\sum_j \Omega_j f_j) - 1} + (1-\lambda)\frac{\beta_k \mu}{\Omega_k} = 0. \tag{A.30}$$

Summing over k yields

$$1 - \sum_j \Omega_j f_j = \frac{\lambda + \frac{\mu-1}{\mu}(1-\lambda)}{\lambda + (\mu-1)(1-\lambda)},$$

and (A.30) yields

$$\Omega_k f_k = \beta_k \frac{(\mu-1)(1-\lambda)}{\lambda + (\mu-1)(1-\lambda)}. \tag{A.31}$$

Using (A.31), the condition (A.29) can be rewritten as

$$\frac{\mu\lambda}{\lambda + \mu - 1} < \phi_1. \tag{A.32}$$

Step 2. Assume that only EC1 binds. From Lemma C.5, we have $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1}$, and Ω_2 is given by (A.19).

Using (A.25), condition (A.20) can be rewritten as

$$\frac{(1-\lambda)\beta_1}{\mu\beta_2(1-\lambda)+1} < A(1-\xi(\phi_1)). \tag{A.33}$$

Step 3. Conditions (A.32) and (A.33) are necessary for the corresponding solution to be optimal. To establish that they are also sufficient, we show that (A.33) cannot hold if (A.32) does. That is, if a solution where only PC1 binds is feasible, then a solution where only EC1 binds is not, and vice versa. Since the only other possibility (when both PC 1 and EC 1 binds) imposes an extra constraint relative to either of these, that solution cannot be optimal when one of these less constrained solutions is feasible.

Suppose that (A.32) holds. This is equivalent to $A < A^* \equiv (1 - \frac{\mu\lambda}{\lambda+\mu-1})\beta_1$. Lemma C.6 shows that $\frac{\partial\xi}{\partial A} < 0$, therefore we also have $A(1-\xi(A)) < A^*(1-\xi(A^*))$. To establish that (A.33) cannot hold, it is thus sufficient to show that $A^*(1-\xi(A^*)) < \frac{(1-\lambda)\beta_1}{\mu\beta_2(1-\lambda)+1}$. Rewrite this as

$$1 - \xi(A^*) < \frac{\lambda+\mu-1}{(\mu-1)(\mu\beta_2(1-\lambda)+1)}.$$

Plugging $A = A^*$ into (A.23) it can be directly verified that this condition holds.

Step 4. Suppose $A = \beta_1$ (i.e., $\phi_1 = 0$). Plugging into (A.23) and solving, it is easily verified that (A.33) holds. From the result in Step 3 and Lemma C.6, it follows that there must be some $\tilde{A} \in [A^*, \beta_1)$ such that (A.33) holds if and only if $A > \tilde{A}$. Equivalently, there exists $\widetilde{\phi}_1 \in (0, \frac{\mu\lambda}{\lambda+\mu-1}]$ such that (A.33) holds if and only if $\phi_1 < \widetilde{\phi}_1$.

Step 5. The only remaining case is when neither (A.32) nor (A.33) holds. Then it must be that both PC1 and EC1 bind: $\Omega_1 f_1 = \mu A(1 - \sum_j \Omega_j f_j)$, from which

$$\Omega_1 f_1 = \frac{\mu A(1 - \Omega_2 f_2)}{1 + \mu A}.$$

Substituting into the objective, the first order condition w.r.t. Ω_2 yields (A.28). This is a quadratic equation, and it can be verified that only the lower root satisfies $\pi_2 \geq f_2$. ■

Using the above characterizations, we turn to the propositions stated in the text.

Proof of Proposition 3. From Proposition C.1, $\Omega_1 = \frac{\beta_1}{f_1} \frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1} < \frac{\beta_1}{f_1} = \Omega_1^{FE}$. ■

Proof of Proposition 4. Part 1. Consider first the cases characterized in Propositions C.1 and C.2.
In case (i) of Proposition C.1, $\Omega_1 = \Omega_2$ when $\beta_1 = \beta_2$ and $f_1 = f_2$.
In case (ii) of Proposition C.1, we need to show that $\frac{\beta(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1} \geq \Omega_2$, where Ω_2 is given by (A.19). Using algebra, this can be simplified to

$$\lambda \frac{\mu + 1}{\mu - 1} \geq \phi_1. \tag{A.34}$$

Notice that, since PC1 and EC2 are both slack, case (ii) of Proposition C.2 also applies. Therefore we must have $\phi_1 < \tilde{\phi}_1 < \frac{\mu\lambda}{\lambda+\mu-1}$, and since $\lambda\frac{\mu+1}{\mu-1} > \frac{\mu\lambda}{\lambda+\mu-1}$, (A.34) is true.

In case (iii) of Proposition C.1, we need to show that $\frac{(1-\lambda)\mu}{(1-\lambda)(\mu-1)+1} \geq \frac{\mu(\lambda+(1-\lambda)(1+\mu\beta))}{(\frac{1}{1-\phi_2}+\mu\beta)(\lambda+\mu-\lambda\mu)}$. Using algebra, this simplifies to $\phi_2 \geq \lambda$, which is exactly one of the conditions for case (iii).

In case (i) of Proposition C.2, $\Omega_1 = \Omega_2$.

As shown above, case (ii) of Proposition C.2 corresponds to case (ii) of Proposition C.1 for which we have already shown that $\Omega_1 > \Omega_2$.

In case (iii) of Proposition C.2, we need to show that $\beta\frac{\mu(1-\phi_1)(1-\Omega_2)}{1+(1-\phi_1)\mu\beta} \geq \Omega_2$ or, equivalently, $\frac{\mu A}{1+2A\mu} \geq \Omega_2$, where Ω_2 is given by (A.28). Using algebra, this can be simplified to $\frac{\lambda\mu}{\lambda+\mu-1} \geq \phi_1$, which is exactly one of the conditions for case (iii).

Finally note that, for $\phi_1 < \phi_2$, the only case not covered by Propositions C.1 and C.2 is when both EC and PC bind in both industries. This would mean $\Omega_j = \mu\frac{\beta}{f}(1 - \phi_j)(1 - \sum_j \Omega_j f)$, from which

$$\Omega_j = \frac{\beta}{f} \frac{1 - \phi_j}{1 - \beta\phi_1 - \beta\phi_2} \tag{A.35}$$

and it immediately follows that $\Omega_1 > \Omega_2$.

Part 2. Notice that, by switching 1 and 2 when needed, Propositions C.1 and C.2 characterize all possible solutions except when both EC and PC bind in both industries. In this last case, $\Omega_j = \frac{\beta_j}{f_j}\frac{1-\phi_j}{1-\beta_1\phi_1-\beta_2\phi_2}$ (see (A.35)). Taking derivatives shows that $\frac{\partial\Omega_1}{\partial\phi_1} < 0$ and $\frac{\partial\Omega_2}{\partial\phi_1} > 0$. For the cases covered in Propositions C.1 and C.2, we can prove the proposition by showing that $\frac{\partial\Omega_k}{\partial\phi_j} \leq 0$ and $\frac{\partial\Omega_j}{\partial\phi_j} \geq 0$ for *both* $j = 1, 2$, $j \neq k$.

In Proposition C.1, Ω_1 is unaffected by ϕ_1 or ϕ_2. In case (i), Ω_2 is also unaffected by ϕ_1 and ϕ_2. In case (ii), Lemma C.6 implies that $\frac{\partial \Omega_2}{\partial \phi_1} > 0$, while ϕ_2 has no effect. In case (iii), Ω_2 is unaffected by ϕ_1, and, taking the derivative, $\frac{\partial \Omega_2}{\partial \phi_2} > 0$.

In Proposition C.2, the number of firms is unaffected by ϕ_2. In case (i), the number of firms is also unaffected by ϕ_1. Case (ii) corresponds to case (ii) of Propositions C.1, and the same argument applies. In case (iii), from (A.28), $\frac{\partial \Omega_2}{\partial A}$ is proportional to $\mu\left(\mu-1\right) f_2\Omega_2 - \mu\beta_2$. Using Ω_2, algebra shows that this is negative, implying $\frac{\partial \Omega_2}{\partial \phi_1} > 0$. From $\Omega_1 = \frac{\beta_1}{f_1}\frac{(1-\phi_1)(1-\Omega_2 f_2)}{1+(1-\phi_1)\mu\beta_1}$, we get $\frac{\partial \Omega_1}{\partial \phi_1} < 0$. ■

D Appendix: Sanctions (proofs for Section 4)

Proof of Proposition 5. A only enters the leader's problem (11) as a constant lowering W. ■

Proof of Propositions 6 and 7. Lemmas D.7-D.9 below give a complete characterization of the solution of the problem with sanctions and $J = 1$, along with comparative statics on W and Y_L. Propositions 6 and 7 in the main text highlight the results corresponding to the oligarchy regime, that is, those in Lemma D.9 . ■

Lemma D.7 *Let*

$$\lambda'\left(\phi, B, C\right) = \frac{\left(\bar{\beta}-1\right)\mu\left(\bar{\beta}(\phi(B+C+1)-C)-B\right)}{\bar{\beta}^2\mu(\phi(B+C+1)-C)-\bar{\beta}(\mu B(\phi+1)+C(\mu+1)(\phi-1))+B\mu+(\phi-1)(C+1)}$$

If $\lambda < \lambda'\left(\phi, B, C\right)$, *then* $\Omega_j = \frac{\beta_j}{f_j}\frac{(1-\lambda)\mu\left(\bar{\beta}-B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta})+1\right)\left(\bar{\beta}(1-\lambda)\mu+\lambda\right)}$, *and*

$$\frac{\partial \ln Y_L}{\partial B} = \frac{\bar{\beta}-1}{\bar{\beta}-B(1-\bar{\beta})}$$
$$\frac{\partial \ln Y_L}{\partial C} = 0$$

and

$$\frac{\partial \ln W}{\partial B} = \mu\bar{\beta}\frac{\bar{\beta}-1}{\bar{\beta}-B(1-\bar{\beta})}$$
$$\frac{\partial \ln W}{\partial C} = \mu\bar{\beta}\frac{\bar{\beta}-1}{C(1-\bar{\beta})+1}.$$

Proof. Following the logic of the baseline case, when λ is small, only (16) binds, hence

$$w_j = \left(1+C\right) f_j$$

and using (5), problem (15) simplifies to

$$\max_{\Omega} \lambda \ln Y_L(\Omega, B, C) + (1-\lambda) \sum_j \left(\beta_j \mu \left(\ln \Omega_j - \ln \frac{c_j + A}{\mu - 1} \right) \right)$$

where

$$Y_L(\Omega, B, C) = \frac{\bar{\beta}\left(1 + B + C \sum_j f_j \Omega_j\right) - \left(B + (1 + C) \sum_j f_j \Omega_j\right)}{1 - \bar{\beta}}. \tag{A.36}$$

The first order conditions give the solution

$$\Omega_j = \frac{\beta_j}{f_j} \frac{(1-\lambda)\mu\left(\bar{\beta} - B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta}) + 1\right)\left(\bar{\beta}(1-\lambda)\mu + \lambda\right)} \tag{A.37}$$

And the implied profit is

$$\frac{\pi_j}{f_j} = \frac{\beta_j \mu \left(1 - \bar{\beta}\right)(1-\lambda)(B + C + 1) + \lambda\left(C\left(1 - \bar{\beta}\right) + 1\right)}{\left(1 - \bar{\beta}\right)(1-\lambda)\mu\left(\bar{\beta} - B(1-\bar{\beta})\right)}$$

Solving $\frac{\pi_j}{f_j} = \frac{1}{1-\phi}$ for λ gives $\lambda'(\phi, B, C)$. Substituting (A.37) into (A.36) and differentiating yields $\frac{\partial \ln Y_L}{\partial B}$ and $\frac{\partial \ln Y_L}{\partial C}$ as stated.

Substituting (A.37) into the welfare function and differentiating gives

$$\begin{aligned}
\frac{\partial \ln W}{\partial B} &= \frac{\partial}{\partial B} \sum_j \beta_j \mu \left(\ln \frac{\beta_j}{f_j} \frac{(1-\lambda)\mu\left(\bar{\beta} - B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta}) + 1\right)\left(\bar{\beta}(1-\lambda)\mu + \lambda\right)} \right) = -\mu\bar{\beta} \frac{1 - \bar{\beta}}{\bar{\beta} - B\left(1 - \bar{\beta}\right)} \\
\frac{\partial \ln W}{\partial C} &= \frac{\partial}{\partial C} \sum_j \beta_j \mu \left(\ln \frac{\beta_j}{f_j} \frac{(1-\lambda)\mu\left(\bar{\beta} - B(1-\bar{\beta})\right)}{\left(C(1-\bar{\beta}) + 1\right)\left(\bar{\beta}(1-\lambda)\mu + \lambda\right)} \right) = -\mu\bar{\beta} \frac{1 - \bar{\beta}}{C\left(1 - \bar{\beta}\right) + 1}.
\end{aligned}$$

■

Lemma D.8 *Let*

$$\lambda''(\phi, B, C, D) = \frac{\left((1-\phi) - \mu\left(1 - \bar{\beta}\right)\right)\left(\left(\phi\bar{\beta} - 1\right)B + \bar{\beta}(\phi - 1)(C + D) + \bar{\beta}\right)}{\left(\phi\bar{\beta} - 1\right)\left((1-\phi) - \mu\left(1 - \bar{\beta}\right)\right)B + (\phi - 1)\left(\bar{\beta}\left(-\mu - \phi + \mu\bar{\beta}\right) + 1\right)C + \mu\bar{\beta}\left(\bar{\beta} - 1\right)(1 - D(1-\phi))}$$

If $\lambda'(\phi, B, C) < \lambda < \lambda''(\phi, B, C, D)$, *then* $\Omega_j = \frac{\beta_j}{f_j} \frac{1-\phi}{1-\phi\bar{\beta}}$, *and*

$$\begin{aligned}
\frac{\partial \ln Y_L}{\partial B}\Big|_{C=0} &= -\frac{1 - \phi\bar{\beta}}{(1 + B)\phi\bar{\beta} - B} \\
\frac{\partial \ln Y_L}{\partial C}\Big|_{B=0} &= -\frac{1 - \phi}{\phi - C(1 - \phi)} \\
\frac{\partial \ln W}{\partial B} &= \frac{\partial \ln W}{\partial C} = 0.
\end{aligned}$$

Proof. Similarly as in the baseline model, we can consider the Lagrangian

$$\max_{\Omega_1} \lambda \ln Y_L(\Omega, B, C) + (1-\lambda) \sum_j \left(\beta_j \mu \left(\ln \Omega_j - \ln \frac{c_j + A}{\mu - 1} \right) \right) + \sum_j \gamma_{PC}^j \left(w_j - (1+C) f_j \right)$$
$$+ \sum_j \gamma_{EC}^j \left(w_j - \left(D(1-\phi)\pi_j + C f_j \right) \right)$$

and conjecture that both (17) and (16) bind. This implies

$$w_j = (1+C) f_j = D(1-\phi)\pi_j + C f_j$$

which, using (5) implies

$$\Omega_j = \frac{\beta_j}{f_j} \frac{1-\phi}{1-\phi\bar{\beta}}.$$

Then, calculating the implied γ_{PC}^j and γ_{EC}^j from the first order conditions, the requirement $\gamma_{PC}^j, \gamma_{EC}^j > 0$ gives the condition $\lambda'(\phi, B, C) < \lambda < \lambda''(\phi, B, C, D)$.

Substituting the solution into $Y_L(\Omega_1, B, C)$ and welfare

$$\ln W(\Omega, B, C) = \sum_j \beta_j \mu \left(\ln \Omega_j - \ln \frac{c_j + A}{\mu - 1} \right)$$

and differentiating give the results. ■

Lemma D.9 *If $\lambda''(\phi) < \lambda$, then*

$$\Omega_j = \frac{\beta_j}{f_j} \frac{(1-\lambda)\mu}{\lambda \frac{-(1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}} - C}{(1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}}(1-X) - CX - B} - (1-\lambda)\frac{X}{1-X}}$$

where $X \equiv \sum_k \Omega_k f_k$ is the lower root of the quadratic equation

$$-X\lambda \frac{(1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}} + C}{(1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}}(1-X) - CX - B} - \frac{(1-\lambda)X}{1-X} + (1-\lambda)\mu\bar{\beta} = 0 \qquad \text{(A.38)}$$

Moreover,

$$\lim_{B \to 0} \frac{\partial \ln Y_L|_{C=0,D=1}}{\partial B} = -\frac{\left(1-\bar{\beta}\right)\left(1 + (1-\lambda)^2 \mu \bar{\beta}\right)}{\phi \bar{\beta}}$$
$$\lim_{C \to 0} \frac{\partial \ln Y_L|_{B=0,D=1}}{\partial C} = -\frac{(1-\lambda)^2 \left(1-\bar{\beta}\right)\mu}{\phi}$$
$$\lim_{D \to 1} \frac{\partial \ln Y_L|_{B=C=0}}{\partial D} = -\frac{1-\phi}{\phi},$$

and

$$\begin{aligned}
\lim_{B\to 0}\frac{\partial \ln W|_{C=0,D=1}}{\partial B} &= -\frac{\lambda^2\left(1-\bar{\beta}\right)\mu}{\phi}\\
\lim_{C\to 0}\frac{\partial \ln W|_{B=0,D=1}}{\partial C} &= -\frac{\lambda^2\left(1-\bar{\beta}\right)\mu}{\phi}\\
\lim_{D\to 1}\frac{\partial \ln W|_{B=C=0}}{\partial D} &= \frac{(1-\phi)\,\bar{\beta}}{1-\bar{\beta}\phi}.
\end{aligned}$$

Proof. Following the logic of the baseline case, if λ is large, only (17) binds, hence

$$w_j = D\,(1-\phi)\,\pi_j + Cf_j$$

and problem (15) simplifies to

$$\max_{\Omega}\lambda\ln Y_L\left(\Omega,B,C,D\right)+(1-\lambda)\left(\ln I\left(\Omega,B,C,D\right)+\sum_j\beta_j\mu\left(\ln\Omega_j-\ln\frac{c_j+A}{\mu-1}\right)\right)$$

where, using (5),

$$\begin{aligned}
Y_L\left(\Omega,B,C,D\right) &= \sum_j\Omega_j\left(\pi_j-D\left(1-\phi\right)\pi_j-Cf_j\right)-B = \qquad\qquad (A.39)\\
&= (1-(1-\phi)D)\frac{\bar{\beta}}{1-\bar{\beta}}(1-X)-CX-B
\end{aligned}$$

and $I\left(\Omega,B,C,D\right)$ denotes workers' and clients' aggregate income:

$$\begin{aligned}
I\left(\Omega,B,C,D\right) &= \sum_j\left(1+\Omega_j\left(D\left(1-\phi\right)\pi_j-f_j\right)\right) =\\
&= (1-X)\left(1+(1-\phi)D\frac{\bar{\beta}}{1-\bar{\beta}}\right)
\end{aligned}$$

Taking the first order conditions with respect to Ω_j yields

$$-f_j\lambda\frac{(1-\phi)D\frac{\bar{\beta}}{1-\bar{\beta}}+C}{Y_L}-f_j\frac{1-\lambda}{1-X}+\frac{(1-\lambda)\beta_j\mu}{\Omega_j}=0$$

Summing over $j=1,...,J$ yields the quadratic equation (A.38). It can be verified that the solution is the larger root (the lower root yields $Y_L<0$). Expressing Ω_j from (D) yields the expression in the statement.

We substitute Ω_j into (A.39), differentiate and take the limit to get the derivatives with respect to the sanction parameters.

Similarly, using

$$\ln W(\Omega, A, B, C, D) = \ln I(\Omega, B, C, D) + \sum_j \beta_j \mu \left(\ln \Omega_j - \ln \frac{c_j + A}{\mu - 1} \right)$$

we differentiate with respect to the sanction parameters and take the limit to obtain the expressions in the statement.

Comparing the derivatives of W and Y_L verifies the statements in Propositions 6 and 7. ∎

E Appendix: Public procurement

E.1 Proofs for Section 5

Proof of Proposition 8. Here we provide a full solution for the case of $J = 1$. (See Appendix E.2 for the solution and additional results for the $J = 2$ case.) From (18), the quantity of the public good is

$$Q_J = q_J \Omega_J^{\mu} = \frac{\tau(Y - \Omega_J(\pi_J - w_J)) + \Delta}{mc_J} \Omega_J^{\mu-1}. \tag{A.40}$$

and consumption of the numeraire good is

$$Q_0(i) = \frac{\hat{\beta}_0}{1 - \hat{\beta}_J} (1 - \tau) Y(i),$$

Substituting in Q_J and Q_0 into the welfare function gives the modified problem

$$\max_{\Omega_J, w_J, m, \tau} \lambda \ln (\Omega_J(\pi_J(\Omega_J) - w_J)) + \\ (1-\lambda) \left[\begin{array}{c} \left(1 - \hat{\beta}_J\right) \ln (1-\tau)(1 + \Omega_J(w_J - f_J)) + \hat{\beta}_J \ln (\tau (1 + \Omega_J(w_J - f_J)) + \Delta) \\ + \hat{\beta}_J (\mu - 1) \left(\ln \Omega_j - \frac{\ln c_j}{\mu - 1} \right) - \hat{\beta}_J \ln m \end{array} \right] \tag{A.41}$$

subject to

$$\pi_J(\Omega_J) = (\tau(1 + \Omega_J(w_J - f_J)) + \Delta) \frac{m-1}{m\Omega_J}$$

(8) and (9).

Following the argument for the baseline case in Proposition 2, consider first the case when only the PC (8) binds, hence $w_J = f_J$. Then, the first order conditions give

$$\begin{aligned}
\tau &= 1 - (1+\Delta)(1-\lambda)\frac{1-\hat{\beta}_J}{1+\hat{\beta}_J(\mu-1)(1-\lambda)} \\
\Omega_J &= (1+\Delta)\frac{(\mu-1)\hat{\beta}_J}{f_J}\frac{(1-\lambda)}{1+\hat{\beta}_J(\mu-1)(1-\lambda)} \\
m &= \frac{\lambda}{(1-\lambda)\hat{\beta}_J} + \mu.
\end{aligned}$$

substituting in the above expressions into (A.40) gives

$$Q_J = \frac{f^{\mu-1}\left(\frac{(\Delta+1)(1-\lambda)(\mu-1)\hat{\beta}_J}{(1-\lambda)(\mu-1)\hat{\beta}_J+1}\right)^{\mu}}{\mu-1}. \tag{A.42}$$

For the EC to be slack, we need $\frac{\pi_J(\Omega_J)}{f_J} < \frac{1}{1-\phi}$, which yields

$$\lambda < \lambda^{PP\prime} = \frac{(\mu-1)\phi\hat{\beta}_J}{(\mu-1)\phi\hat{\beta}_J - \phi + 1}$$

Now consider the case when only the EC (9) binds, implying $w_J = (1-\phi)\pi_J$. The first order conditions give

$$\begin{aligned}
\tau &= 1 - \frac{(1+\Delta)\phi\left(1-\hat{\beta}_J\right)(1-\lambda)}{(\Delta+1)\lambda(1-\phi) - \Delta(1-\lambda)(\mu-1)\phi\hat{\beta}_J + \phi}, \\
\Omega_J &= \frac{(\mu-1)\hat{\beta}_J}{f_J}\frac{(\Delta+1)(1-\lambda)}{(1-\lambda)(\mu-1)\hat{\beta}_J+1}, \\
m &= \frac{\lambda}{(1-\lambda)\phi\hat{\beta}_J} + 1,
\end{aligned}$$

implying (A.42) again for Q_J.

For the PC to be slack, we need $\frac{\pi_J(\Omega_J)}{f_J} > \frac{1}{1-\phi}$ or $\lambda > \lambda^{PP''}$. We find that $\lambda^{PP''} = \lambda^{PP\prime} \equiv \lambda^{PP}$, that is, there is no Constrained industry capture range in this case.

Part 2 of the proposition is easily verified by differentiating the expressions above. ■

Proof of Proposition 9. The first statement follows from substituting in the solutions from the proof of Proposition 8 into W and Y_L and taking derivatives.

For the second statement, we solve problem (A.41) with the additional, binding constraint of $m = \bar{m}$ (and setting $\Delta = 0$). For small λ, (i.e. binding (8)), this gives

$$\tau = \frac{\lambda + (1-\lambda)\mu\hat{\beta}_J}{(1-\lambda)(\mu-1)\hat{\beta}_J + 1}$$

$$\Omega_J = \frac{\bar{m}-1}{\bar{m}} \frac{1}{f_J} \frac{(1-\lambda)(\mu-1)\hat{\beta}_J \left((1-\lambda)\mu\hat{\beta}_J + \lambda\right)}{(\mu-1)\hat{\beta}_J(1-\lambda)\left((1-\lambda)(\mu-1)\hat{\beta}_J + (\lambda+1)\right) + \lambda}$$

It is easy to check that $\bar{m}$ binds iff

$$\hat{\beta}_J(\bar{m}-\mu) < \frac{\lambda}{1-\lambda}$$

Substituting into (A.40) for this case gives

$$Q_J = \left(\frac{(1-\lambda)\mu\hat{\beta}_J + \lambda}{\bar{m}}\right)^{\mu} \frac{\left(\frac{\bar{m}-1}{f} \frac{(1-\lambda)(\mu-1)\hat{\beta}_J}{(\mu-1)\beta_J\left((1-\lambda)^2(\mu-1)\hat{\beta}_J + 1 - \lambda^2\right) + \lambda}\right)^{\mu-1}}{(1-\lambda)(\mu-1)\hat{\beta}_J + 1}. \tag{A.43}$$

For large λ, (i.e. binding (9)) we get

$$\tau = \frac{\bar{m}\left(\lambda + (1-\lambda)\hat{\beta}_J\right)}{1 + (\bar{m}-1)\left(\lambda + (1-\lambda)\left(\phi + (1-\phi)\,\hat{\beta}_J\right)\right)},$$

$$\Omega_J = \frac{(1-\lambda)(\mu-1)\hat{\beta}_J}{f_J\left((1-\lambda)(\mu-1)\hat{\beta}_J + 1\right)}$$

and $\bar{m}$ binds if and only if

$$\phi\hat{\beta}_J\,(\bar{m}-1) < \frac{\lambda}{1-\lambda}.$$

In this case, we have

$$Q_J = \frac{f^{\mu-1}\left(\lambda + (1-\lambda)\hat{\beta}_J\right)\left(\frac{(1-\lambda)(\mu-1)\hat{\beta}_J}{(1-\lambda)(\mu-1)\hat{\beta}_J + 1}\right)^{\mu}}{(\mu-1)\,(1-\lambda)(1 + (\bar{m}-1)\phi)\hat{\beta}_J}. \tag{A.44}$$

Then, substituting into W and Y_L and differentiating our expressions give the results. ■

E.2 Public procurement with multiple industries

In this appendix, first we set up the general model with public procurement in sector $J > 1$. Then, we present additional results for the $J = 2$ case, providing further insights on the effect of public procurement compared to our discussion in the main text.

For the general set up, observe that under our assumptions, since consumer i allocates his after-tax income across the goods of the $J-1$ private industries only, his consumption index for industry $j<J$, previously equation (4), changes to

$$Q_j(i) = \frac{\hat{\beta}_j}{1-\hat{\beta}_J}\frac{\Omega_j^{\mu-1}}{\mu c_j}(1-\tau)\,Y(i).$$

Consumption of the numeraire good is

$$Q_0(i) = \frac{\hat{\beta}_0}{1-\hat{\beta}_J}(1-\tau)\,Y(i),$$

and given the leader's choice, the quantity of the public good is

$$Q_J = q_J\Omega_J^{\mu} = \frac{\tau(Y-\sum_{j>0}\Omega_j(\pi_j-w_j))}{mc_J}\Omega_J^{\mu-1}. \tag{A.45}$$

The leader's problem is now

$$\max_{\Omega,w,m,\tau} \lambda\ln\left(\sum_j \Omega_j(\pi_j-w_j)\right)+(1-\lambda)\left[\begin{array}{c}\ln\left(1+\sum_j\Omega_j(w_j-f_j)\right)+\sum_{j>0}\beta_j\mu\left(\ln\Omega_j-\frac{\ln c_j}{\mu-1}\right)\\ +(1-\hat{\beta}_J)\ln(1-\tau)+\hat{\beta}_J(\ln\tau-\ln m)\end{array}\right] \tag{A.46}$$

subject to (6),(8),(9) and profit expressions that modify (5):

$$\pi_j = \frac{\beta_j}{\left(1-\hat{\beta}_J\right)\Omega_j}\left(\sum_{j>0}\Omega_j(\pi_j-w_j)+(1-\tau)(Y-\sum_{j>0}\Omega_j(\pi_j-w_j))\right) \tag{A.47}$$

$$\pi_J = \tau(Y-\sum_{j>0}\Omega_j(\pi_j-w_j))\frac{m-1}{m\Omega_J}. \tag{A.48}$$

The following propositions describe the benchmark case of a welfare-maximizing leader ($\lambda=0$), the equilibrium when $\lambda>0$, and finally the impact of external transfers. We first state the propositions, then prove them jointly.

Proposition E.3 *Suppose that $\lambda=0$ and $J=2$. The leader does not limit entry, $\pi_1=f_1$, $\pi_J=f_J$, and does not distort the markup, $m=\mu$. However, the tax revenue is used to increase spending in industry J leading to more competition in industry J and less spending and less competition in every other industry:*

$$\Omega_J = \Omega_J^{PP} \equiv \frac{\beta_J}{f_J(1-\beta_0)} > \Omega_J^{FE}$$

and

$$\Omega_1 = \Omega_1^{PP} \equiv \frac{\beta_1}{f_1}\frac{\left(1-\beta_0-\hat{\beta}_J\right)}{(1-\beta_0)\left(1-\hat{\beta}_J\right)} < \Omega_1^{FE}$$

and $\tau = \tau \equiv \frac{\hat{\beta}_J}{1-\beta_0} > \hat{\beta}_J$.

Proposition E.4 *Suppose that* $J = 2$, $\phi_j = \phi\ \forall j$. *Then there exist thresholds* λ'_P *and* λ''_P *such that:*
1. (Industry capture) When $\lambda \in (0, \lambda'_P)$, $w_1 = f_1$, $w_J = f_J$. *In addition, for* $\lambda < \min\left(\lambda'_P, \frac{(\mu-1)\hat{\beta}_0}{1+(\mu-1)\hat{\beta}_0}\right)$ *the leader does not limit entry in industry 1,* $\pi_1 = f_1$, *therefore, he does not extract profit from this sector.*
2. (Oligarchy) When $\lambda \in (\lambda''_P, 1)$, $w_1 > f_1$, $w_J > f_J$ *and* $\Omega_j = \frac{\beta_j \mu}{f_j}\frac{(1-\lambda)}{\mu\beta(1-\lambda)+1}\ \forall j$. *Still, as* $m > \mu$, $\frac{\pi_J}{f_J} > \frac{\pi_1}{f_1} > 1$.
In both cases the leader overprices public goods, $m > \mu$, *overspends on public procurement,* $\tau > \hat{b}^{PP}$, *and limits entry in the public good sector,* $\Omega_J < \Omega_J^{PP}$, *to generate profit* $\pi_J > f_J$. *Furthermore, as* λ *increases, each of these distortions increases in magnitude (i.e.,* m, τ *and* π_J *increase while* Ω_J *decreases), and the quantity of public goods,* Q_J, *falls.*

Proposition E.5 *Suppose that* $J = 2$, $\phi_j = \phi$ *and we are at an interior solution (*$\lambda \in (0, \lambda'_P)$ *or* $\lambda \in (\lambda''_P, 1)$ *where* $\lambda'_P < \lambda''_P$ *is determined in Proposition E.4). Let* $\Omega_j(\Delta)$, $\pi_j(\Delta)$, $m(\Delta)$, *etc. denote equilibrium outcomes for a given* Δ.

1. *Then markups, profits and the relative number of firms across industries are insensitive to* Δ, *i.e.,* $\pi_j(\Delta) = \pi_j(0)$, $m(\Delta) = m(0)$, *and* $\frac{\Omega_1(\Delta)}{\Omega_2(\Delta)} = \frac{\Omega_1(0)}{\Omega_2(0)}$.

2. *The number of firms and each industry's revenue is proportional to* $(1+\Delta)$, *i.e., for* $\forall j$

$$\Omega_j(\Delta) = (1+\Delta)\,\Omega_j(0)$$
$$P_j(\Delta)Q_j(\Delta) = (1+\Delta)\,P_j(0)Q_j(0)$$

3. *Hence, a reduction in external funds reduce both welfare and the leader's income. The relative effect is given by*

$$\frac{\partial \ln W}{\partial \Delta} \Big/ \frac{\partial \ln Y_L}{\partial \Delta} = [\mu(1-\hat{\beta}_0) + \hat{\beta}_0] > 1.$$

Proof of Propositions E.3, E.4 and E.5. We solve the Lagrangian corresponding to problem (A.46) but with the budget constraint (18) that includes external funds. For Proposition E.4, set $\Delta = 0$ in the expressions below and for Proposition E.3, set $\lambda = 0$ as well.

1. (Industry capture) Assume that both PCs bind and both are ICs slack. That is $w_1 = f_1$, $w_J = f_J$, and $\frac{\pi_j}{f_j} < \frac{1}{1-\phi}$. Then there are two possible subcases.

(i) Suppose that $\pi_1 > f_1$. Then solving (A.46), we obtain

$$
\begin{aligned}
\Omega_j &= (1+\Delta)\frac{\mu\beta_j}{f_j}\frac{1-\lambda}{\lambda+(1-\lambda)\mu(1-\beta_0)} \text{ for } j=1,J \\
m &= \frac{(\mu-1)\hat{\beta}_0\left(\left(1-\hat{\beta}_0\right)(\mu-1)+\hat{\beta}_J\right)+\left(1-\hat{\beta}_J\right)\mu\hat{\beta}_J}{\hat{\beta}_J\left(\hat{\beta}_0(\mu-1)+1-\hat{\beta}_J\right)}+\frac{\lambda}{(1-\lambda)\hat{\beta}_J} \\
\tau &= 1-\mu\left(1-\hat{\beta}_J\right)^2(1-\lambda)\frac{1+\Delta}{\left((\mu-1)\hat{\beta}_0+1-\hat{\beta}_J\right)\left(\lambda+\left(\mu\left(1-\hat{\beta}_0\right)+\hat{\beta}_0\right)(1-\lambda)\right)} \\
\frac{\pi_1}{f_1} &= \frac{(\lambda-1)\hat{\beta}_J+1}{(1-\lambda)\left(\hat{\beta}_0(\mu-1)-\hat{\beta}_J+1\right)} \\
\frac{\pi_J}{f_J} &= \frac{\hat{\beta}_0\left(1-\hat{\beta}_0\right)(\mu-1)+\left(1-\hat{\beta}_J\right)\hat{\beta}_J}{\hat{\beta}_J\left(\hat{\beta}_0(\mu-1)+1-\hat{\beta}_J\right)}+\frac{\hat{\beta}_0(\mu-1)\lambda+\left(1-\hat{\beta}_J\right)\lambda}{(1-\lambda)(\mu-1)\hat{\beta}_J\left(\hat{\beta}_0(\mu-1)+1-\hat{\beta}_J\right)}
\end{aligned}
$$

Then $\pi_1 > f_1$ is equivalent to $\frac{\hat{\beta}_0(\mu-1)}{\hat{\beta}_0(\mu-1)+1} < \lambda$.

(ii) Suppose that $\frac{\hat{\beta}_0(\mu-1)}{\hat{\beta}_0(\mu-1)+1} > \lambda$, in which case we must have $\pi_1 = f_1$. From (A.46), we then get

$$
\begin{aligned}
\Omega_1 &= \frac{\beta_1}{f_1}\frac{(\Delta+1)\left((1-\lambda)\left(\mu\hat{\beta}_1+\hat{\beta}_0\right)+\lambda\right)}{\left(1-\hat{\beta}_J\right)\left((1-\lambda)\left(\mu\left(1-\hat{\beta}_0\right)+\hat{\beta}_0\right)+\lambda\right)} \\
\Omega_J &= \frac{(\Delta+1)(1-\lambda)(\mu-1)\hat{\beta}_J}{f_J\left((1-\lambda)\left(\mu\left(1-\hat{\beta}_0\right)+\hat{\beta}_0\right)+\lambda\right)} \\
m &= \frac{\lambda\left(\left(1-\hat{\beta}_0\right)(\lambda+\mu(1-\lambda))+\hat{\beta}_0\right)+(1-\lambda)^2\mu\left(1-\hat{\beta}_J\right)\hat{\beta}_J}{(1-\lambda)\hat{\beta}_J\left(1-(1-\lambda)\hat{\beta}_J\right)} \\
\tau &= 1-\left(1-\hat{\beta}_J\right)(1-\lambda)(\Delta+1)\frac{\lambda+(1-\lambda)\left(\mu\hat{\beta}_1+\hat{\beta}_0\right)}{\left(1-\hat{\beta}_J(1-\lambda)\right)\left(\lambda+(1-\lambda)\left(\mu\left(1-\hat{\beta}_0\right)+\hat{\beta}_0\right)\right)} \\
\frac{\pi_J}{f_J} &= \frac{(1-\lambda)\hat{\beta}_J\left(1-\hat{\beta}_J\right)-\hat{\beta}_0\lambda}{\hat{\beta}_J\left(1-(1-\lambda)\hat{\beta}_J\right)}+\lambda\frac{\lambda+\mu(1-\lambda)-\hat{\beta}_J(1-\lambda)}{\hat{\beta}_J(1-\lambda)\left(1-\hat{\beta}_J(1-\lambda)\right)(\mu-1)}
\end{aligned}
$$

For Lemma E.3, set $\lambda = 0$ and $\Delta = 0$ in the expressions above. For the first and second parts of Proposition E.5, observe that both in cases (i) and (ii) π_1, π_J and m are independent of Δ while Ω_1, Ω_J are proportional to $(1+\Delta)$. This also implies that expenditures $P_1Q_1 = \mu c\Omega_1$ and $P_JQ_J = mc\Omega_J$ are proportional to $(1+\Delta)$.

We also have to find the threshold λ'_P such that for any $\lambda < \lambda'_P$ the IC constraints for both industries are slack as conjectured. Note that $\frac{\pi_1}{f_1} < \frac{\pi_J}{f_J}$, because $m > \mu$ and $\frac{\pi_J}{f_J}$ is monotonically increasing in λ. Therefore, we only need to ensure that $\lambda < \lambda'_P$ implies $\frac{\pi_J}{f_J} < \frac{1}{1-\phi}$. For $\lambda = 0$, we have $\frac{\pi_J}{f_J}|_{\lambda=0} = 1$ (from case (ii)). In addition, in both cases $\lim_{\lambda\to 1}\frac{\pi_J}{f_J} = \infty$. Therefore, by continuity, there must be a λ'_P for which $\frac{\pi_J}{f_J}|_{\lambda=\lambda'_P} = \frac{1}{1-\phi}$. At the threshold between cases (i) and (ii) we have

$$\frac{\pi_J}{f_J}\Big|_{\lambda=\frac{\hat\beta_0(\mu-1)}{\hat\beta_0(\mu-1)+1}} = \frac{\frac{m-1}{m}(\tau+\Delta)}{f_J\Omega_J}\Big|_{\lambda=\frac{\hat\beta_0(\mu-1)}{\hat\beta_0(\mu-1)+1}} = \frac{\hat\beta_0\left(\frac{\mu}{\hat\beta_J}-1\right)+1-\hat\beta_J}{1+(\mu-1)\hat\beta_0-\hat\beta_J}$$

therefore, if

$$\frac{\hat\beta_0\left(\frac{\mu}{\hat\beta_J}-1\right)+1-\hat\beta_J}{1+(\mu-1)\hat\beta_0-\hat\beta_J} < \frac{1}{1-\phi}$$

then λ'_P is given by the $\lambda \in (0,1)$ solving $\frac{\pi_J}{f_J} = \frac{1}{1-\phi}$ in the first subcase.

2. (Oligarchy) Now assume that λ is sufficiently large that only the IC constraints bind, that is $\frac{\pi_j}{f_j} > \frac{1}{1-\phi}$ for both $j = 1, J$. Then, from (A.46) we get

$$\begin{aligned}
\Omega_j &= (1+\Delta)\,\tfrac{\hat\beta_j(\mu-1)}{f_j}\tfrac{1-\lambda}{1+(1-\lambda)(\mu-1)\left(1-\hat\beta_0\right)}\\
m &= \tfrac{1}{\hat\beta_J(1-\lambda)}\tfrac{\left(1-\hat\beta_J\right)\left(\frac{\lambda}{\phi}+\hat\beta_J(1-\lambda)\right)-\beta_1}{1-\hat\beta_J-\beta_1}\\
\tau &= 1-\tfrac{\mu\phi\left(1-\hat\beta_J\right)^2(1-\lambda)(\Delta+1)}{\left(\left(1-\hat\beta_J\right)\left(\phi\left(1-\hat\beta_0+\mu\hat\beta_0\right)(1-\lambda)-\mu(\phi-\lambda)\right)-\phi\hat\beta_0(\mu-1)^2\left(1-\hat\beta_0\right)(1-\lambda)\right)\Delta+\left(\phi\hat\beta_0(\mu-1)+\left(1-\hat\beta_J\right)(\phi+\lambda\mu(1-\phi))\right)}\\
\frac{\pi_1}{f_1} &= \tfrac{1-(1-\lambda)\hat\beta_J}{(1-\lambda)\left(\hat\beta_0(\mu-1)+1-\hat\beta_J\right)}\\
\frac{\pi_J}{f_J} &= \tfrac{\left(1-\hat\beta_J\right)\lambda\mu-(\mu-1)\phi\left(1-(1-\lambda)\hat\beta_J\right)\left(1-\hat\beta_0-\hat\beta_J\right)}{(1-\lambda)(\mu-1)\phi\hat\beta_J\left(\hat\beta_0(\mu-1)+1-\hat\beta_J\right)}
\end{aligned} \tag{A.49}$$

As in case 1, for the first and second parts of Proposition E.5, observe that π_1, π_J and m are independent of Δ while Ω_1, Ω_J are proportional to $(1+\Delta)$. This also implies that expenditures $P_1Q_1 = \mu c\Omega_1$ and $P_JQ_J = mc\Omega_J$ are also proportional to $(1+\Delta)$.

We also have to find the threshold λ''_P such that for any $\lambda > \lambda''_P$ the IC constraints for both industries bind as conjectured. Note that $\frac{\pi_1}{f_1} < \frac{\pi_J}{f_J}$, because $m > \mu$ and that $\frac{\pi_1}{f_1}$ is monotonically increasing in λ. Therefore, we only need to ensure that $\lambda > \lambda''_P$ implies $\frac{\pi_1}{f_1} > \frac{1}{1-\phi}$. It is easy to see that

$$\lambda''_P \equiv 1 - \frac{1}{\left(\hat\beta_0(\mu-1)+1-\hat\beta_J\right)\frac{1}{1-\phi}+\hat\beta_J}$$

is the solution.

Differentiating the above expressions for m, τ, π_J, and Ω_J w.r.t. λ verifies the statements of Proposition E.4. Substituting them into the expressions for welfare and the leader's income Y_L, and directly calculating the elasticities yields Lemma E.5.

Finally, to verify that Q_J is decreasing in λ (Proposition E.4), we calculate Q_J using

(A.45) and the expressions above for each case (with Δ set to 0). This yields

$$\begin{aligned} Q_J &= \frac{\hat{\beta}_J^{\mu}}{c_J} \frac{1}{\frac{\lambda}{1-\lambda} + \mu(1-\hat{\beta}_0) + \hat{\beta}_0} \left(\frac{1}{f_J} \frac{\mu - 1}{\frac{\lambda}{1-\lambda} + \mu(1-\hat{\beta}_0)} \right)^{\mu-1} \\ Q_J &= \frac{(\mu-1)^{\mu-1}}{c_J f_J^{\mu-1}} \left(\frac{\hat{\beta}_J}{\frac{1}{1-\lambda} + (1-\hat{\beta}_0)(\mu-1)} \right)^{\mu} \\ Q_J &= \frac{\hat{\beta}_J^{\mu}}{c_J} \frac{1}{\frac{\lambda}{1-\lambda} + \mu(1-\hat{\beta}_0) + \hat{\beta}_0} \left(\frac{\mu-1}{f_J} \frac{1}{\frac{1}{1-\lambda} + (\mu-1)\left(1-\hat{\beta}_0\right)} \right)^{\mu-1} \end{aligned} \tag{A.50}$$

for cases 1(i), 1(ii), and 2, respectively. The welfare maximizing benchmark, Q_J^{PP} is obtained by setting $\lambda = 0$ in (A.50). It can easily be verified that each of these expressions is smaller than Q_J^{PP} and decreasing in λ. ■

F Derivations for Section 6

F.1 Derivations for Section 6.1 (Innovation)

In the following Proposition we characterize the equilibrium for $\lambda = 0$. The proof also derives in detail the formalization and equations in Section 6.1.

Proposition F.6 *Suppose that there is an a^{IFE} which solves*

$$\frac{\kappa - 1}{\kappa a^{IFE} + (1 - a^{IFE})} = \frac{K'\left(a^{IFE}\right)}{f_j + K\left(a^{IFE}\right)}$$

then

1. *Under free entry, firms choose innovation intensity a^{IFE}, and $\Omega_j = \Omega_j^{IFE} \equiv \frac{\beta_j}{f + K(a^{IFE})}$.*
2. *If $\lambda = 0$, the leader's optimal choices of $\Omega_j = \Omega^{IFE}$, $\bar{w}_j = \bar{\pi}_j$ and $\underline{w}_j = \underline{\pi}_j$ implement the innovation intensity a^{IFE}.*

Proof of Proposition F.6. Consider an entrepreneur who takes Ω_j, P_j and Y as given. His problem is

$$\max_{\bar{p}_j(\omega), \underline{p_j}(\omega), a} \hat{\beta}_j Y P_j^{\sigma-1} \left(a \bar{p}_j(\omega)^{-\sigma} \left(\bar{p}_j(\omega) - \frac{c_j}{\hat{\kappa}} \right) + (1-a) \underline{p_j}(\omega)^{-\sigma} \left(\underline{p_j}(\omega) - c_j \right) \right) - K(a)$$

giving the first order conditions

$$\bar{p}_j(\omega) = \mu \frac{c_j}{\hat{\kappa}} \tag{A.51}$$

$$\underline{p_j}(\omega) = \mu c_j \tag{A.52}$$

and

$$\hat{\beta}_j Y P_j^{\sigma-1} (\mu - 1) (c_j)^{1-\sigma} \mu^{-\sigma} (\kappa - 1) = K'(a). \tag{A.53}$$

If all entrepreneurs choose effort level a' and prices (A.51)-(A.52) conditional on success or failure, then, by the law of large numbers, a' fraction of firms succeed and offer $\bar{p}_j(\omega)$ for each variety ω. Hence, the price index is

$$P_j^{\sigma-1} = \left[\int_0^{\Omega_j} p(\omega)^{1-\sigma} d\omega\right]^{-1} = \left[a'\Omega_j \bar{p}_j(\omega)^{1-\sigma} + (1-a')\,\Omega_j \underline{p_j}(\omega)^{1-\sigma}\right]^{-1} = \tag{A.54}$$
$$= \frac{(\mu c_j)^{\sigma-1}}{(\kappa a' + (1-a'))\,\Omega_j}.$$

Substituing into (A.53) and simplifying gives the first order condition

$$\beta_j Y \frac{\kappa - 1}{(\kappa a' + (1-a'))\,\Omega_j} = K'(a). \tag{A.55}$$

In equilibrium, $a' = a$, which, along with (A.54) and prices (A.51)-(A.52) give profits (19)-(20).

In expectation, entrepreneurs' profit is

$$\tilde{\pi}(\Omega, a) \equiv a\bar{\pi}_j(\Omega_j, a) + (1-a)\,\underline{\pi}_j = \frac{\beta_j Y}{\Omega_j} - K'(a). \tag{A.56}$$

Then the zero profit condition and (6) implies $Y = 1$ and gives the expression

$$\Omega_j^{IFE} \equiv \frac{\beta_j}{f_j + K(a)}. \tag{A.57}$$

Substituting back into (A.55) gives a^{FE} as the solution of

$$\frac{\kappa - 1}{\kappa a + (1-a)} = \frac{K'(a)}{f_j + K(a)}. \tag{A.58}$$

This proves the first statement of the Proposition.

For the leader's problem, observe first that the quantity index in industry j modifies to

$$Q_j = \left[a\Omega_j \left(\hat{\beta}_j Y P_j^{\sigma-1} \bar{p}_j(\omega)^{-\sigma}\right)^{\frac{1}{\mu}} + (1-a)\,\Omega_j \left(\hat{\beta}_j Y P_j^{\sigma-1} \underline{p_j}(\omega)^{-\sigma}\right)^{\frac{1}{\mu}}\right]^{\mu} =$$
$$= (\kappa a + (1-a))^{\mu-1}\,\Omega_j^{\mu-1} \hat{\beta}_j Y (\mu_j c_j)^{-1}.$$

Using $\tilde{w}_j(a) = a\bar{w}_j + (1-a)\,\underline{w}_j$, the objective function of the leader (after omitting constants) is

$$\lambda \ln \Sigma_{j>0} \Omega_j (\tilde{\pi}(\Omega) - \tilde{w}_j(a)) +$$
$$(1-\lambda) \ln (\Sigma_{j>0} \Omega_j (\tilde{w}_j(a) - f_j) + 1) + \Sigma_{j>0} \beta_j \mu \ln \Omega_j - \Sigma_{j>0} \beta_j \frac{\mu}{\mu - 1} \ln c_j + \Sigma_{j>0} \beta_j \mu \ln (\kappa a + (1-a)).$$

The leader maximizes this objective by choosing $\bar{w}_j, \underline{w}_j, \Omega_j$ for each j subject to (A.56),

$$Y = \Sigma_{j>0}\Omega_j \left(\tilde{\pi}\left(\Omega, a\right) - f_j\right) + 1, \tag{A.59}$$

the participation constraint

$$\tilde{w}_j\left(a\right) \geq f_j, \tag{A.60}$$

and (21)-(23). Recall that the leader cannot directly choose a, instead it is determined by (21).

When $\lambda = 0$, the leader will not extract any income, hence $\bar{w}_j = \bar{\pi}_j\left(\Omega_j, a\right)$ and $\underline{w}_j = \underline{\pi}_j\left(\Omega_j, a\right)$, which implies that (22) and (23) trivially holds. Then, for any fixed a, the derivation for choosing the optimal Ω_j is analogous to our proof in Proposition 1, implying $Y = 1$ and $\Omega_j = \Omega_j^{IFE}$. But for these choices,

$$\begin{aligned}\bar{w} - \underline{w} = \bar{\pi}_j\left(\Omega^{FE}, a\right) - \underline{\pi}_j\left(\Omega^{FE}, a\right) &= \frac{\hat{\beta}_j Y}{\Omega_j}\frac{\mu - 1}{\mu}\frac{\kappa - 1}{a\kappa + (1-a)} \\ &= \left(f_j + K\left(a\right)\right)\frac{\kappa - 1}{\kappa a + (1-a)}\end{aligned}$$

which, from (A.58), is equal to $K'\left(a\right)$ when $a = a^{FE}$, making (21) hold. This concludes the proof of the second statement in the Proposition. ■

F.2 Derivations for Section 6.2 (Favoring clients)

Proof of Proposition 10. After taking logs and dropping the constants, the leader's objective function becomes

$$\max_{\Omega,w} \lambda \ln\left(\sum_j \Omega_j(\pi_j - w_j)\right) + (1-\lambda)\left[\ln\left(1 + \sum_j \Omega_j(\alpha_j w_j - f_j)\right) + \sum_j \beta_j \mu \ln \Omega_j\right] \tag{A.61}$$

1. When $\lambda = 0$, $w_j = \pi_j$, so (A.61) becomes

$$\max_{\Omega} \ln\left(1 - \sum_j \Omega_j f_j + \sum_j \alpha_j \Omega_j \pi_j\right) + \sum_j \beta_j \mu \ln \Omega_j.$$

But since $\Omega_j \pi_j$ is proportional to $1 - \sum\limits_j \Omega_j f_j$ (see (A.4) and (A.5)), this is equivalent to $\max_\Omega \ln\left(1 - \sum\limits_j \Omega_j f_j\right) + \sum_j \beta_j \mu \ln \Omega_j$. The leader's problem, and therefore the solution, is the same as in the $\alpha_j = 1$ case.

2. Suppose that only the EC constraints bind, so that $w_j = (1 - \phi_j)\pi_j(\Omega)$. Then (A.61) can be written as

$$\max_{\Omega} \lambda \ln\left(\sum_j \Omega_j \pi_j \phi_j\right) + (1-\lambda)\left[\ln\left(1 - \sum_j \Omega_j f_j + \sum_j \alpha_j (1-\phi_j)\Omega_j \pi_j\right) + \sum_j \beta_j \mu \ln \Omega_j\right].$$

Just as in part 1, because $\Omega_j \pi_j$ is proportional to $1 - \sum\limits_j \Omega_j f_j$, the α_j terms do not affect the maximization problem, which simply becomes

$$\max_{\Omega} \ln\left(1 - \sum_j \Omega_j f_j\right) + (1-\lambda)\sum_j \beta_j \mu \ln \Omega_j.$$

Again the solution is the same as in the $\alpha_j = 1$ case.

The only case in which the equivalence with the $\alpha_j = 1$ case breaks down is when the clients' payment w_j does not depend on profits. This is the case when only the PC constraints bind so that $w_j = f_j$. Now (A.61) becomes

$$\max_{\Omega} \lambda \ln\left(\overline{\beta} - \sum_j \Omega_j f_j\right) + (1-\lambda)\left[\ln\left(1 + \sum_j \Omega_j f_j(\alpha_j - 1)\right) + \sum_j \beta_j \mu \ln \Omega_j\right].$$

The first-order conditions for $j = 1, ..., J$ can be written as

$$F^j(\Omega, \alpha) \equiv \frac{-f_j \lambda}{\overline{\beta} - \sum\limits_k \Omega_k f_k}\Omega_j + \frac{(1-\lambda) f_j}{\frac{1}{\alpha_j - 1} + \sum\limits_k \Omega_k f_k \frac{\alpha_k - 1}{\alpha_j - 1}}\Omega_j + (1-\lambda)\beta_j \mu = 0. \tag{A.62}$$

Use F_x^j to denote the derivative of F^j w.r.t. x. For $J = 1$, $sign\frac{\partial \Omega_1}{\partial \alpha} = sign F_\alpha^1$ as $F_{\Omega_1}^1 < 0$ (the second-order condition for maximization). From (A.62), $F_\alpha^1 = \frac{\partial}{\partial \alpha}\left(\frac{(1-\lambda) f_1}{\frac{1}{\alpha - 1} + \Omega_1 f_1}\right)$, therefore $\frac{\partial \Omega_1}{\partial \alpha} > 0$ and as α increases above 1, Ω_1 rises. Proceeding as in the proof of Proposition 2 2 it can be verified that $\lambda'(\phi)$ increases.

Next, consider the $J = 2$ case. Using Cramer's rule and the second-order condition for maximization, we get that

$$sign\frac{\partial \Omega_1}{\partial \alpha_1} = sign\left(-F_{\alpha_1}^1 F_{\Omega_2}^2 + F_{\Omega_2}^1 F_{\alpha_1}^2\right).$$

The second-order condition implies that $F_{\Omega_2}^2 < 0$, and from (A.62), we get $F_{\Omega_2}^1 < 0$, $F_{\alpha_1}^1 > 0$ and $F_{\alpha_1}^2 < 0$. Thus, $\frac{\partial \Omega_1}{\partial \alpha_1} > 0$.

Similarly,

$$sign\frac{\partial \Omega_2}{\partial \alpha_1} = sign\left(-F_{\Omega_1}^1 F_{\alpha_1}^2 + F_{\alpha_1}^1 F_{\Omega_1}^2\right).$$

Again, $F_{\Omega_1}^1 < 0$ from the second-order condition, $F_{\alpha_1}^1 > 0$, and $F_{\Omega_1}^2 < 0$, which implies $\frac{\partial \Omega_2}{\partial \alpha_1} < 0$. ■

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