Sustainable Investing and Public Goods Provision*

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Abstract

We model investors that take into account the amount of public good that firms produce (e.g., by reducing carbon emissions) when making their portfolio allocation. In an equilibrium asset pricing model with production and public goods provision, we find that environmentally conscious investors invest more than others, invest more in clean firms, and may invest more in dirty firms. Whether clean firms exhibit CAPM alphas depends on the amount of systematic risk of the firm and its relative contribution to the public good. There is underprovision of the public good in equilibrium. Lower government provision may lead to a surge in investment and government provision may be dominated by green subsidies. Finally, we extend the model to analyze negative externalities, donations, and uncertainty regarding public good provision.

Keywords: Sustainable finance, ESG investing, public good provision, asset pricing

JEL Codes: G11, G12, H41

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1 Introduction

Global sustainable investment has reached $35.3 trillion (GSIA, 2020). The rise of sustainable investing comes coupled with the question of what effect it has. Do investors make a difference? Do their sustainability concerns affect firms' cost of capital and lead to better societal outcomes? How does sustainable investment compare to and interact with taxes and subsidies in achieving impact?

We examine these issues by modeling sustainable investment using an approach that is standard in public economics: we posit that people care about the provision of public goods. When the government provides insufficient levels of public goods, the private provision of public goods arises. While it is typical to think of such provision through nonprofits, firms may also provide public goods - and the reduction of their negative externalities may also be thought of as contributions to the public good. The financial markets, through the targeting of investments, may influence firm decision-making, and hence their contributions to the public good - this is the thesis for sustainable finance.

To fix ideas, we will think of the public good as the mitigation of climate change. We consider investors who take into account that their investments may provide both financial risks and returns as well as the provision of the public good. These investors may be large institutions, such as pension funds or sovereign wealth funds. Firms maximize profits, deciding on their capital allocation between production and investment in mitigating climate change (e.g., by modifying their production processes). We study these choices in an equilibrium asset pricing model with production and public goods provision.

Investors who are more environmentally conscious invest more wealth in the financial market in equilibrium than investors who care less about the public good (who we name financial investors). The environmentally conscious investors always invest more in clean firms, and, when the correlation between the clean and dirty firms is low, the environmentally conscious investor will also invest more in dirty firms than the financial investor, for hedging purposes. Compared to the conventional CAPM, if a stock's relative environmental investment contribution is larger (smaller) than its systematic risk, the stock price carries a premium (discount) and the CAPM alpha is negative (positive). This implies that if a clean firm holds a lot of systematic risk, it may also outperform in terms of expected returns. And even if a clean firm underperforms, investors are better off as they enjoy the public good it provides.

We demonstrate that despite the concern about public good provision, there is a standard free riding effect; investors do not internalize the benefit of their investments on others, resulting in the underprovision of the public good relative to a social planner's choice.

Government provision of the public good (through taxation and spending) will crowd out private provision of the public good. However, unlike the literature on crowding out (e.g., Bergstrom, Blume, and Varian (1986)), we find that crowding out may be partial (less than 100%) or excessive (more than 100%). In the latter case, increased government provision reduces the total provision of the public good. Excessive crowd out may occur when there are large investments in clean firms already, since taxes reduce these investments and may be replaced with (possibly inefficient) government provision.

These results on crowding out also, of course, imply that if government provision decreases, private provision will increase. Therefore they could be related to the surge in sustainable investment just after the surprise election of Donald Trump as U.S. president in 2016: the expected decrease in governmental support for the environment could have prompted investors to react by compensating for the shortfall. Indeed, Ramelli et al. (2021) find that firms with a high level of climate responsibility have a high abnormal return right after the surprise 2016 election of Donald Trump.[1,2] Therefore,

[1]Ramelli et al (2021) posit something akin to a crowding out effect as well as a boomerang effect, where Trump-policies are expected to be reversed shortly after his presidency.

[2]This is consistent with evidence in the mutual fund space, where inflows into ESG mutual funds in the first half of 2017 were $3.5 billion (compared with $4.9 billion for the whole of 2016). "The Trump White House gave these mutual funds a big boost," *For-*

government inaction might explain part of the rise of sustainable finance.

We also allow for the government to provide a green subsidy. The subsidy has a tradeoff: it reduces the cost of green investment, but, because it is funded by taxation, it reduces overall investment. Therefore we find that the green subsidy may first increase welfare, but as it becomes too large, it can lead to welfare losses. This is because large green subsidies crowd out the private good production. We also compare the optimal subsidy to the optimal tax used to fund public good provision. Despite the subsidy's drawbacks, it performs better in simulations than the optimal tax. This is due to the fact that the optimal tax is zero when environmentally conscious investors' preferences are not strong.

We look at several extensions to the model:

Firstly, we examine the case where the dirty firm contributes negatively to the public good, i.e. we allow for negative externalities. Here, there are two results that diverge from our main model. We show that in this case the environmental investor always invests less in the dirty firm than the financial investor. Surprisingly, despite the negative contribution of the dirty firm, this model may have a larger provision of the public good relative to the main model, because investment becomes concentrated in the clean firm.

Secondly, our results are robust to allowing investors to contribute directly to the public good via donations, e.g., to a charity or NGO. The tradeoff in donating is giving up financial returns in exchange for more public good and less portfolio risk. Allowing for donations weakly increases provision of the public good - we find that unless the preference for the public good is particularly intense, investors don't donate given the possibility of investing.

Lastly, we introduce uncertainty in how public good contributions result in benefits. We have in mind that the benefits of climate mitigation strategies are not completely known. We show that more public good uncertainty

tune.com, June 16, 2018. Catherine Woodman, a CFP and principal advisor at Impact Investors noted, "Clients want to use every avenue they can to push back on the presidential administration." "Trump creates positive effect on socially responsible investing," by Sonya Dreizler, *cnbc.com*, April 23, 2019.

is related to lower public good production and that the environmental investor invests less in the clean firm as the correlation between the clean firm's productivity and the public good provision increases.

Our paper is complementary to Oehmke and Opp (2020) and Besley and Ghatak (2007), where agents take into account their effect on the amount of public good provided and care about the public good regardless of whether they help provide it. Oehmke and Opp (2020) study corporate financing rather than asset pricing. Besley and Ghatak (2007) study the influence of the product market on the provision of public goods.

Pastor et al. (2021) study a related model in which infinitesimally small investors have nonpecuniary benefits/warm glow from holding green stocks.[3] In an extension, they allow firms to choose their scale and their green-ness; already green (brown) firms become more green (less brown) through the cost of capital channel. The tilt in investment from brown to green leads to higher social impact. In contrast, we assume that investors are large and take into account both the effect and size of their investments.

Our model shows that the CAPM alpha is positive when firm's systematic risk is larger than its relative contribution to green investment, and vice versa. In our numerical analysis, the CAPM alpha is generally positive for dirty firms and negative for clean firms (although we have also identified a small number of cases when alphas flip). This is consistent with empirical literature that has documented investors seeking compensation for holding brown firms (e.g., Bolton and Kacperczyk (2021, 2022) Huij et al (2022), and Hsu et al (2022)). Pastor et al. (2022) estimate lower expected returns for green stocks than for brown.[4]

A key aspect of the model is that the investors care about the provision of the public good. There is little direct evidence on how much institutional

[3]In Goldstein et al. (2021), investors also care about a nonpecuniary element, but the focus is on information flow between investors. There are several equilibrium models of ESG investing where investors exogenously screen out dirty firms or give extra weight to clean firms: Baker et al. (2020), Betermier et al. (2023), Heinkel et al. (2001), and Luo and Balvers (2017).

[4]On the other hand, Pastor et al. (2022) find higher realized returns for green firms (and lower for carbon-intensive firms) which they attribute to unexpected strong increases in environmental concerns. Zhang (2023) also finds the lack of a carbon premium.

investors care about sustainability.[5] Krueger et al. (2020) show in a survey that institutional investors care about climate change and its associated risks. Chen et al. (2021) show that firms with more institutional ownership perform better on CSR metrics. Bolton and Kacperczyk (2021) demonstrate that insurance companies, investment advisers, and pension funds significantly divest from firms associated with higher scope 1 emissions.

In Section 2, we describe the model. In Section 3, we solve for the equilibrium. In Section 4, we explore the asset pricing implications of the model. In Section 5, we show that there is underprovision of the public good and examine government provision of the public good, both directly and through firm subsidies. In Section 6, we extend the model in three directions: allowing for negative externalities from the dirty firm, allowing for donations, and making the provision of the public good uncertain/risky. Section 7 concludes. All proofs and simulations are in the appendix.

2 The Model

The model has two dates, $t = 0$ and $t = 1$. At $t = 0$, companies sell shares to investors. Investors may invest their initial wealth in these shares and in the riskless asset. The companies allocate their capital to their normal production and to climate change mitigation (the public good). At $t = 1$, production occurs, the state of the world realizes, and investors enjoy payoffs from their investments and the public good. We now provide more details on investors and firms.

2.1 Investors

There are two investors, an environmentally conscious investor E and a financial investor F. We will think of these investors as institutions such as pension funds, large hedge funds, and sovereign wealth funds. Such institu-

[5] There are several papers on *individual* investors, showing that they behave more like they have warm glow preferences, i.e., their investments are tilted towards cleaner firms, but the tilt does not relate to the amount of impact the firm has (Heeb et al. (2021), Bonnefon et al. (2022), and Riedl and Smeets (2017)).

tions vary in their preferences depending on their stakeholders, mandates, and fiduciary responsibilities. In 2020, 75% of sustainable investing was done by institutional investors (GSIA (2020)).

For simplicity, both investors i ($i \in (E, F)$) start with an endowment of initial wealth ω_0 at date $t = 0$. Investors derive utility from terminal wealth - we use the Merton (1987) mean-variance utility function to model this. In addition, we assume that the investors have an additional term in their utility, $f(G)$, that represents their value from public good consumption. The term G is the total amount of public good, and f is an increasing and concave function, i.e. $f'(\cdot) > 0, f''(\cdot) \leq 0$. Investor i's preference is given as follows:

$$U_i = \mathbb{E}_0(\tilde{\omega}_i) - \nu Var(\tilde{\omega}_i)/(2\omega_0) + \psi_i f(G),$$

where $\tilde{\omega}_i$ denotes her stochastic terminal wealth, ν is a risk aversion coefficient, and ψ_i expresses how much investors care about the public good, which differs among investors. We assume that environmentally conscious investors care more about the public good than financial investors, i.e., $\psi_E > \psi_F$.

Investor i uses her initial wealth to purchase firms' shares from the firms and to trade a riskless asset at $t = 0$. The riskless asset is in zero net supply but each firm's stock is in positive net supply. At date $t = 1$, the state of nature is realized, and the investor's terminal wealth is realized and the public good is provided. Let r_n be the stochastic[6] return of firm n and $\tilde{r}_n$ be the return of firm n in excess of the risk-free rate r_f, i.e., $\tilde{r}_n = r_n - r_f$. There will be two types of firms (C and D, described below) and the correlation between the two types' returns is ρ (< 0).

Let $\theta_{i,n}$ be the fraction of investor i's wealth invested in firm n and $\theta_{i,f}$ be the fraction of her wealth invested in the risk-free asset. Her budget constraints at $t = 0$ and $t = 1$ are as follows:

[6] We will define its distribution in the next subsection.

$$\sum_{n\in(C,D)} \theta_{i,n}\omega_0 + \theta_{i,f}\omega_0 = \omega_0,$$

$$\tilde{\omega}_i = \sum_{n\in(C,D)} (1+r_n)\theta_{i,n}\omega_0 + (1+r_f)\theta_{i,f}\omega_0.$$

We can combine the two budget constraints to express the investor's terminal wealth at $t = 1$ as:

$$\tilde{\omega}_i = \omega_0 \left(1 + r_f + \sum_{n\in(C,D)} \tilde{r}_n\theta_{i,n} \right), \tag{1}$$

We define market portfolio weights as $\theta_C = \frac{1}{2}\sum_{i\in(E,F)} \theta_{i,C}$ and $\theta_D = \frac{1}{2}\sum_{i\in(E,F)} \theta_{i,D}$.

We now characterize the maximization problem of the environmentally conscious investor E. The financial investor's optimization is the same except she places a lower weight (ψ_F) on the public good. Moreover, investors are sophisticated: they internalize the effect of their own portfolio choice on public goods provision, i.e., $G = H(\Theta_E, \Theta_F)$, where $\Theta_E \equiv [\theta_{E,C}, \theta_{E,D}]'$ and $\Theta_F \equiv [\theta_{F,C}, \theta_{F,D}]'$ denote the vector of investor's portfolio allocations to the clean and dirty firms, and H maps the portfolio choices to a quantity of public good (and will be determined by the equilibrium).

Investor E solves the following problem to decide on asset allocations:

$$\underbrace{\max}_{\Theta_E} \; U_E = \mathbb{E}_0(\tilde{\omega}_E) - \nu Var(\tilde{\omega}_E)/(2\omega_0) + \psi_E f(G),$$

where $\tilde{\omega}_E = \omega_0 \left(1 + r_f + \sum_{n\in(C,D)} \tilde{r}_n\Theta_E\right)$ and $G = H(\Theta_E, \Theta_F)$.

2.2 Firms

There are two firms that produce private goods. In order to do so, they choose the amount of capital to allocate to production and to climate change mitigation (the public good), assuming that they take as given the cost of capital.

We allow firms to vary in their relative productivity of climate change mitigation. Climate change mitigation investments may reduce the long term costs of energy transition[7]. Note that firms may also benefit from climate change mitigation through increased demand for their goods from more satisfied customers or increased productivity from (i) more satisfied/healthier workers (e.g., Edmans (2011))[8], (ii) improved relationships with their suppliers, or (iii) improvement in production processes. In these examples, the investment in climate change mitigation affects both the profitability of the firm and the public good. Therefore, for simplicity, we will bundle these benefits into the production process. We also allow for no benefits from climate change mitigation.

Firms' private goods production function is Cobb-Douglas. Let k_n be the total investment of firm n and z_n be the amount dedicated to climate change mitigation. Therefore the amount of investment for production is $k_n - z_n$. Let $\tilde{\epsilon}_n$ be the productivity shock to the production of private goods, which is normally distributed. The production technology is represented by:

$$\tilde{y}_n = \tilde{\epsilon}_n (k_n - z_n)^{\gamma_n} z_n^{1-\gamma_n},$$

where γ_n is the parameter that characterizes the trade-off between production and climate change mitigation. The two types of firms are denoted by $n = D, C$. Dirty firms D are carbon intensive, while clean firms C find it easier and/or more profitable to reduce carbon emissions. We thus assume that $\gamma_C < \gamma_D \leq 1$.

Firms' total investments in climate change mitigation contribute to the public good through a linear transformation, i.e., $G = \sum_n z_n$.

We make a few remarks regarding the production function. First, note that when $\gamma_D = 1$, climate change mitigation is only costly for the dirty firm and has zero benefits. Second, note that even when the parameter γ_n is less than 1, the marginal product of climate change mitigation z_n may be

[7]Due to regulatory risk or changes in carbon pricing (as in Bustamante and Zucchi (2023)).

[8]Similarly, along the "social" dimension of ESG, another example would be workplace safety (less injuries means more productivity for the firm *and* lower burden on society).

positive or negative. Lastly, we point out that in Section 6.1, we modify the model to explore the situation where the dirty firm's input of z_n decreases the amount of public good.[9]

The excess financial return of firm n stock is simply $\tilde{r}_n = \frac{\tilde{y}_n}{k_n} - 1 - r_f$, with the risk-free rate endogenously determined in equilibrium. We denote the mean of $\tilde{r}_n$ as $\bar{r}_n$ and the variance as σ^2. This distribution of $\tilde{r}_n$ is implied by the normally distributed productivity shock and is thus Gaussian as well.

Firms maximize contingent claim value for the shareholders, so in the firms' objective function, their monetary profits are adjusted by the stochastic discount factor, denoted by $\tilde{M}$, which is determined in equilibrium through state prices. The fact that shareholders enjoy utility from both financial returns and the public good is in equilibrium reflected in $\tilde{M}$. Therefore the contingent claim value for shareholders and firms' objective functions indirectly reflect the market's preference for both the private good and the public good. Of course, since the amount of capital firms invest must be equal to the amount of capital offered by investors, the cost of capital directly affects their choices in equilibrium.

Firm n's maximization decision is given by:

$$\underbrace{\max}_{z_n, k_n} \; E_0 \tilde{M} \tilde{\epsilon}_n (k_n - z_n)^{\gamma_n} z_n^{1-\gamma_n} - k_n,$$

and firm n's investment decisions yield (see proof in Appendix A.1):

$$z_n = (1 - \gamma_n) k_n. \tag{2}$$

Firm n's investment in the public good is determined by the amount of capital k_n, which is determined in equilibrium, and the firm's relative preference for the public good γ_n, which is an exogenous characteristic of the firm. Substituting Equation (2) into the production function, we obtain $\tilde{y}_n = \tilde{A}_n k_n$, where $\tilde{A}_n = \tilde{\epsilon}_n \gamma_n^{\gamma_n} (1 - \gamma_n)^{1-\gamma_n}$.

[9]Note that our formulation is related to the dynamic macro literature on growth and taxation with carbon externalities (Acemoglu et al. (2012), Barrage (2020)). There, individuals are affected negatively by climate change which is affected by firm production decisions.

3 Equilibrium

We now solve for the equilibrium allocation. The equilibrium consists of the investors' portfolio allocations (Θ_E, Θ_F), firm investments (k_C, k_D, z_C, z_D), and stock expected excess returns $\bar{r}_C, \bar{r}_D$ such that:

(i) the investors' portfolios maximize their expected utility;

(ii) each firm chooses investments to maximize contingent claim value for shareholders;

(iii) asset markets clear;

$$k_n = \sum_i \theta_{i,n} \omega_0, \tag{3}$$

$$\sum_i \theta_{i,f} = 0, \tag{4}$$

(iv) payoffs are realized and distributed;

$$\sum_i \tilde{\omega}_i = \sum_n \tilde{\epsilon}_n (k_n - z_n)^{\gamma_n} z_n^{1-\gamma_n}, \tag{5}$$

(v) and the public good is produced:

$$G = \sum_n z_n. \tag{6}$$

Given the equilibrium definition, we can demonstrate that the derivative of the public good with respect to investor portfolio choices is linear in initial wealth, and solve for the multiplier of initial wealth denoted by η_n, via the chain rule: $\eta_n \omega_0 = \partial G/\partial \theta_{i,n} = \partial G/\partial z_n \cdot \partial z_n/\partial k_n \cdot \partial k_n/\partial \theta_{i,n}$. According to Equation (2), $\partial z_n/\partial k_n = (1 - \gamma_n)$, and the market clearing conditions for asset markets imply $\partial k_n/\partial \theta_{i,n} = \omega_0$. Given that $\partial G/\partial z_n = 1$, it follows that

$$\eta_{i,n} = 1 - \gamma_n, \tag{7}$$

which only depends on the firm type. With this, we derive the first-order condition for investors' asset holdings, which leads to:

$$\bar{r} = \nu\Sigma\Theta - (1-\gamma)\psi f'(G), \tag{8}$$

where Σ is the covariance matrix, $\psi \equiv \frac{1}{2}\sum_i \psi_i$, $\Theta \equiv [\theta_C, \theta_D]'$ denotes the vector of market portfolio weights, $\bar{r} \equiv [\bar{r}_C, \bar{r}_D]'$ denotes the vector of expected excess returns, and $\gamma \equiv [\gamma_C, \gamma_D]'$. This is proven in Appendix A.2.

In the model setup, we assume that investors purchase shares directly from the firm, and the market clears given their demand and the fixed supply of shares. While we formulate the investors' budget constraints in terms of excess returns, in Appendix A.3 we reformulate the budget constraints in terms of stock prices, and we show these two formulations are equivalent. The stock price formulation shows that higher aggregate capital allocation to the firm leads to a higher stock price.

Rearranging the first-order conditions, the following proposition characterizes the asset allocation in equilibrium.

Proposition 1. *The optimality conditions for investor i, $i \in \{E, F\}$, lead to the following asset allocation:*

$$\begin{aligned}
\theta_{i,C} &= \frac{\Big(1-\gamma_C-\rho(1-\gamma_D)\Big)\psi_i f'(G) + \bar{r}_C - \bar{r}_D\rho}{\nu(1-\rho^2)\sigma^2}, \\
\theta_{i,D} &= \frac{\Big(1-\gamma_D-\rho(1-\gamma_C)\Big)\psi_i f'(G) + \bar{r}_D - \bar{r}_C\rho}{\nu(1-\rho^2)\sigma^2}
\end{aligned} \tag{9}$$

Since $\psi_E > \psi_F$ and $\gamma_C < \gamma_D$, Equation (9) implies that in equilibrium, investor E invests more in the clean firm than investor F, i.e. $\theta_{E,C} > \theta_{F,C}$. This is natural - investor E receives higher utility from public good provision.

While we are unable to depict the provision of the public good and allocations in completely closed form solutions, as the optimal asset allocations depend on the endogenous asset returns and vice versa, we now do so using a simple numerical example. The base values of the model parameters used in the simulation are provided in the third column of Table 1. Below the table, we describe how these values were chosen. Our figures examine the sensitivity of equilibrium variables to the key model parameters, by varying them one at a time within the ranges set in the fourth column of Table 1, keeping all else equal.

In Figure 1, we examine the provision of the public good. The amount of public good provided increases with both investors' preferences for the public good. As risk aversion increases, public good provision decreases, as investors focus on hedging their portfolios rather than contributing. Furthermore, as the correlation increases, the hedging benefits decrease and both type of investors tilt their portfolios to contribute more to the public good.[10]

In Figure 2, we look at the investment allocations. As the environmentally conscious investor's preference for the public good increases, her allocation to the clean firm increases. However, the financial investor's allocation to the clean firm decreases. This is the free riding effect in action: the financial investor takes advantage of the environmentally conscious investor's investment (resulting in more of the public good) and tilts to increase her returns. We see a similar dynamic play out when the preference of the financial investor for the public good increases, although in this case it is the environmentally conscious investor free riding. We discuss the free riding effect in more detail in the next section. As risk aversion increases, both types of investor hedge by re-allocating towards the dirty firm.

Furthermore, Proposition 1 implies that if the correlation between the returns of the clean stock and the dirty stock is large enough ($\rho > \frac{1-\gamma_D}{1-\gamma_C}$), the environmentally conscious investor invests less in the dirty firm than the financial investor ($\theta_{E,D} < \theta_{F,D}$). Interestingly, if the stock correlation is sufficiently small ($\rho < \frac{1-\gamma_D}{1-\gamma_C}$), investor E invests more in the dirty firm than investor F. This is because investor E has invested more in the clean firm, and, therefore, the diversification benefits for investor E from investing in the dirty firm are appealing. In our numerical illustration, the threshold value for the correlation is equal to $\frac{1-0.9}{1-0.6} = 0.25$, and we can see in the bottom right panel of Figure 2 that indeed the environmental investor has a larger allocation to the dirty firm than the financial investor ($\theta_{E,D} > \theta_{F,D}$)

[10] In In Figure 2, it can be seen that environmental investors tilt at a faster rate. This differential is a free riding effect, which we discuss further in Section 5.1.

when the correlation between the two stocks is lower than 0.25.

We summarize these results in the following corollary.

Corollary 1. *In equilibrium, investor E shorts the riskless asset and investor F longs it. Investor E invests less (more) in the dirty firm than investor F when the stock correlation (ρ) is larger (smaller) than the relative weighting of the dirty and clean firm on the public good ($\frac{1-\gamma_D}{1-\gamma_C}$).*

Proposition 1 also implies that $\theta_{E,f} < 0$ and $\theta_{F,f} > 0$. This means that in equilibrium, investor E borrows from investor F at the risk-free rate to trade stocks. This is because investor E derives more utility from the public goods, and is willing to take on more risk in order to do so. In fact, using the approach of Constantinides (1990), we can show that investors who are environmentally conscious are effectively less risk averse:

Lemma 1. *The absolute risk aversion of environmentally conscious investors is lower than that of financial investors.*

The environmentally conscious investor is less risk averse than the financial investor because public goods act as a stabilizer to financial risks. For example, suppose that ψ_E is extremely large. In this case, the environmental investor cares less about financial risks since she derives a substantial amount of utility from the public good already.[11] Indeed, in deriving the absolute risk aversion, the public good in the utility function reduces its curvature, i.e. the risk aversion - and more weight on the public good therefore means less risk aversion.

4 Asset Pricing Implications

At this point, it is helpful to characterize the asset pricing implications of the model using a standard CAPM model as a benchmark. Premultiplying Equation (8) by Θ' gives the market equilibrium, $\bar{r}_M = \Theta'\bar{r}$, where the subscript M denotes the market:

[11] In subsection 6.3, we extend the model to allow for the provision of the public good to be risky/uncertain, which particularly affects the environmental investor's allocation.

$$\bar{r}_M = \nu\sigma_M^2 - (1 - \sum_n \theta_n\gamma_n)\psi f'(G). \tag{10}$$

We present the new asset pricing expression in the following proposition with the proof in Appendix A.6.

Proposition 2. *The expected excess returns of firm n in equilibrium are:*

$$\bar{r}_n = \beta_n^G \bar{r}_M, \tag{11}$$

where

$$\beta_n^G = \frac{Cov(\tilde{r}_M, \tilde{r}_n) - \nu^{-1}(1-\gamma_n)\psi f'(G)}{\sigma_M^2 - \nu^{-1}(1 - \sum_n \theta_n\gamma_n)\psi f'(G)}. \tag{12}$$

Equation (12) expresses the beta β_n^G that incorporates the public good provision, and the asset pricing formula in (11) resembles the traditional CAPM. Let β_n be the standard CAPM beta, i.e., $\beta_n = \frac{Cov(\tilde{r}_M, \tilde{r}_n)}{\sigma_M^2}$. Using OLS models to regress the excess return of stock n on the excess return of the market, the familiar CAPM equation appears as follows:

$$\bar{r}_n = \frac{Cov(\tilde{r}_M, \tilde{r}_n)}{\sigma_M^2}\bar{r}_M + \alpha_n = \beta_n \bar{r}_M + \alpha_n, \tag{13}$$

where we define the abnormal return α_n as public good factors not captured by the conventional CAPM model.[12] It follows that there will be an abnormal return α_n *iff* $\beta_n \neq \beta_n^G$. We focus on the case when $\bar{r}_M > 0$, which is a natural assumption. This implies that the denominator of β_n^G is positive. Consider the following condition:

$$\beta_n < \frac{1-\gamma_n}{1 - \sum_n \theta_n\gamma_n}. \tag{14}$$

[12] Thus, whenever we refer to abnormal return or alpha in our environment, it only means factors not captured by the conventional CAPM model, and we use alpha or CAPM alpha interchangeably. Specifically, these factors in our environment stem from the public good provision. As a result, alpha may appear as a result of model mis-specification, i.e., using CAPM when there is public goods production, and it does not reflect market inefficiency.

If condition (14) holds, then $\beta_n^G < \beta_n$ and therefore $\alpha_n < 0$. If condition (14) does not hold, then $\beta_n^G \geq \beta_n$ and therefore $\alpha_n \geq 0$.

To see the intuition, we note that the right-hand side of condition (14) represents firm n's public good investment contribution relative to the market, and β_n represents the systematic risk of stock n. If a stock's relative public good investment contribution is larger than its systematic risk, investors demand less compensation for risk but reward more on the stock's contribution to the public good, so the stock price carries a premium and the CAPM-alpha is thus negative. On the flip side, if the stock's systematic risk is larger than its relative public good investment contribution, the investors demand more compensation for risk, so the stock price carries a discount and the abnormal return estimated via CAPM is thus positive.

This implies that even though the clean firm's relative public goods investment contribution is larger than 1 ($\frac{1-\gamma_C}{1-\sum_n \theta_n \gamma_n} > 1$), its stock price can still command a positive alpha if its systematic risk (β_C) is sufficiently large. Thus it is possible for clean stocks to outperform standard benchmarks.

The next proposition expands on this intuition.

Proposition 3. *Whether firm n generates CAPM-alpha depends on its systematic risk β_n and its relative public good investment contribution, i.e.,* $\frac{1-\gamma_n}{1-\sum_n \theta_n \gamma_n}$.

1. *When $\beta_C \leq 1$, the clean firm generates a negative alpha and the dirty firm generates a positive alpha.*

2. *When $\beta_C > 1$, the sign of alpha for each firm depends on the parameters.*

As shown in the proof, the necessary and sufficient condition for $\beta_C \leq 1$ to hold is $\theta_C \leq \theta_D$. Furthermore, whenever $\beta_C \leq 1$, $\beta_D \geq 1$. This means that when clean firms are a smaller part of the market portfolio than dirty firms, clean firms exhibit a systematic risk lower than 1 and dirty firms exhibit a systematic risk higher than 1. Since clean firms' relative public goods investment contribution is higher than that of dirty firms, clean firms generate a negative alpha and dirty firms generate a positive alpha. This delivers what the standard logic in the literature posits (e.g., Pastor et al. (2021), Hong and Kacperczyk (2009)) - sin stocks offer a premium and clean stocks provide lower returns. In those papers, however, the result comes from warm glow preferences or mandates to divest.

However, in our model, we find that these results may reverse - when clean firms constitute a larger part of the market portfolio, it is possible for clean firms' expected returns to exhibit a positive CAPM alpha.

These results may seem counterintuitive. After all, if investors allocate more wealth to clean firms due to preferences for public goods, it should push down the cost of capital for these firms; thus, a negative alpha would be expected. Likewise, if in the aggregate investors allocate less wealth to clean firms, it tends to raise the cost of capital so a positive alpha would be expected. In our environment, this intuition turns out to be untrue: clean firms generate a negative alpha for sure when investors allocate *less* wealth to them in the aggregate. The reason is that the market portfolio itself is affected by the public good provision (see Equation (10)). As less wealth is allocated to the clean firm, its systematic risk is low ($\beta_C < 1$) and investors demand less compensation for risk but reward more for the clean firm's contribution to public goods; hence, a negative alpha arises. If, however, more wealth is allocated to the clean firm, its systematic risk is high ($\beta_C > 1$) and investors demand more compensation for risk: condition (14) may not hold. In this case, the effect on alpha is ambiguous (and for some parameters will be positive).

We examine the conditions under which the CAPM alpha of clean firms may be positive further using our numerical illustration, with the benchmark parameter values once again given in Table 1 and the results presented in Figure 3. As expected, alpha is generally positive for the dirty firm and negative for the clean firm.[13] However, we show that when the correlation

[13]Furthermore, we can observe that in Figure 2, portfolio holdings for dirty firms are generally less than for clean firms. This is broadly consistent with Betermier et al. (2022), who show a negative cross-sectional link between market correlation, i.e. systematic risk, and CAPM alpha. They build an equilibrium model of firm-level capital demand and supply, in which capital supply is exogenously tilted towards firms with specific characteristics, such as good ESG scores or based on subjective beliefs about stock performance.

is sufficiently high, the alpha of clean firms turns positive, albeit slightly. This is because the hedging benefits diminish with the increase in correlation, leading to larger allocations (seen in Figure 2) to the clean firm, which makes it extremely systematic. Investors thus demand more compensation for the risk taken on, pushing alpha for the clean firm to be positive.

Interestingly, Figure 3 also demonstrates that alpha for the dirty firm decreases when risk aversion decreases. In this case, the diversification benefits go down, and both investors allocate less investment to the dirty firm (this can be seen in Figure 2). Firm D's systematic risk therefore is lower.

Pastor et al. (2022) document that green firms have higher realized returns than dirty firms despite having lower expected returns, which they attribute to unexpected strong increases in environmental concerns. In our model, we can proxy for a positive shock in environmental concerns either by increasing the public good preference parameters ψ_E and ψ_F or by increasing the relative weight, i.e. the initial wealth, of the environmental investor. Intuitively, in both cases what we obtain is an increase in the overall allocation to the clean firm, which leads to an increase in its price, consistent with the results in Pastor et al. (2022).[14] At the same time, a positive shock to green demand decreases the price of the dirty firm's shares.

5 Provision of the public good

Is the amount of the public good provided efficient? When agents contribute directly to the public good (as in the classic public economics model), the public good is underprovided as the agents do not take into account the positive externality from their contributions - this is the well known free rider effect. We begin by examining the free rider effect, and then look

In their setting, dispersion in supply generates this effect because firms with low supply of capital tend to be small, have low market correlations, and face high capital costs in equilibrium, thereby yielding positive CAPM alphas. In our setting, the capital supply is endogenously tilted by considering public good production; therefore, the direction of CAPM alpha depends on not only the systematic risk but also the relative public good investment contribution.

[14] Note that firm n's share price, p_n, is given by $p_n = k_n = \sum_{i=E,F} \theta_{i,n}\omega_0$; see details in Appendix A.4.

at whether government provision of the public good through taxation or subsidies improve efficiency.

5.1 Free Riding

In this section, we examine the level of public good provision in our model - where agents contribute through investment - relative to the optimum.

We consider a planner who maximizes the weighted average of the utility of the environmentally conscious investor and the financial investor by choosing their portfolio allocations while respecting the investors' constraints. Since the two investors have equal initial wealth ω_0, we assume equal weights of 1/2 for the two investors as a benchmark. Like investors, the planner is sophisticated: she internalizes the effect of her portfolio choice on public goods provision. The equilibrium that emerges from this planner's problem is the planner's equilibrium. We use superscript p to indicate the planner's choices.

More concretely, the planner solves the following problem to decide on asset allocations:

$$\max_{\{\theta^p_{i,n}\}} \frac{1}{2}\mathbb{E}_0\tilde{\omega}_E + \frac{1}{2}\mathbb{E}_0\tilde{\omega}_F - \frac{\nu}{4\omega_0}\left[Var(\tilde{\omega}_E) + Var(\tilde{\omega}_F)\right] + \frac{1}{2}(\psi_E + \psi_F)f(G),$$

$$s.t. \quad \tilde{\omega}_E = \omega_0 \left(1 + r_f + \sum_{n\in(C,D)} \tilde{r}_n \theta^p_{E,n}\right), \tag{15}$$

$$\tilde{\omega}_F = \omega_0 \left(1 + r_f + \sum_{n\in(C,D)} \tilde{r}_n \theta^p_{F,n}\right), \tag{16}$$

$$G = H(\Theta^p_E, \Theta^p_F), \tag{17}$$

where H maps the portfolio choices to public goods.

The portfolio choice is given in the following proposition.

Proposition 4. *(Planner's equilibrium) The planner allocates assets ac-*

cording to (18)*:*

$$\theta^p_{i,C} = \frac{\Big(1-\gamma_C-\rho(1-\gamma_D)\Big)\sum_i \psi_i f'(G^p) + \bar{r}^p_C - \bar{r}^p_D\rho}{\nu(1-\rho^2)\sigma^2},$$
$$\theta^p_{i,D} = \frac{\Big(1-\gamma_D-\rho(1-\gamma_C)\Big)\sum_i \psi_i f'(G^p) + \bar{r}^p_D - \bar{r}^p_C\rho}{\nu(1-\rho^2)\sigma^2} \tag{18}$$

Investors have identical asset allocations.

Proposition 4 characterizes the planner's equilibrium. Although the two investors value public good differently, in the planner's equilibrium, there is no difference between how they allocate wealth to the clean firm and to the dirty firm. This is because the planner internalizes the benefit of each investment on both investors.

Similar to Proposition 2, we derive the asset pricing implications using the canonical CAPM model as a benchmark in the proposition below. We then use the equilibrium to derive results on the optimal provision of the public good.

Proposition 5. *In the planner's equilibrium, the expected excess returns of firm n are expressed as:*

$$\bar{r}^p_n = \beta^{G^p}_n \bar{r}^p_M, \tag{19}$$

where

$$\beta^{G^p}_n = \frac{Cov(\tilde{r}^p_M, \tilde{r}^p_n) - 2\nu^{-1}(1-\gamma_n)\psi f'(G^p)}{\sigma^2_M - 2\nu^{-1}(1-\sum_n \theta^p_n\gamma_n)\psi f'(G^p)}. \tag{20}$$

Controlling for $\bar{r}$, the planner's equilibrium produces more public goods than the market equilibrium.

The above proposition demonstrates that, absent price effects, the planner's equilibrium has a higher level of public good provision than the market equilibrium. This is because the planner internalizes the benefits of investments on everyone, so the total marginal rate of substitution between the public good and terminal wealth is always higher than that in the market equilibrium. This substitution effect moves wealth allocation from dirty firms to clean firms.

To further understand the intuition, suppose in the planner's equilibrium less of the public good is produced. Then the marginal utility of the public good in the planner's equilibrium would be higher than in the market equilibrium. Since the planner internalizes the benefit of public goods between the two investors, she would allocate more wealth to the clean firm than in the private equilibrium, leading to a higher level of public good, which is a contradiction.

While we are unable to compare the provision of the public good in the planner's and market general equilibrium in closed form solutions, we now do so this using the simple numerical example in Table 1 without assuming away the wealth effect of public goods (that is, we assume $f''(\cdot) < 0$). In the base case, the public good provision is equal to 0.61 in the market equilibrium and 0.71 in the planner's equilibrium, corresponding to about a 16% increase.[15]

Figure 4 shows the sensitivity of the difference in public goods provision to the key model parameters. First, we note that the difference between public good provision in the planner and market equilibria is positive for all parameter configurations. Moreover, $G^P - G$ is economically quite sizeable, ranging between around 0.05 and 0.45, and the percentage increase $(G^P - G)/G$ ranges between 10% and 39%. Second, in general, the relationship between the exogenous parameters and the extent of the free riding problem is non-linear, which reflects the fact that asset allocations are non-linear and non-monotonic in the parameters. However, we can see that $G^P - G$ is increasing in ψ_E and ψ_F. This effect is intuitive, since the severity of the free riding problem increases when either investor cares more about the public good. We see another noticeable pattern in the bottom right panel of Figure 4, which shows the sensitivity of $G^P - G$ to changes in the correlation between the two stocks. As the correlation increases, hedging benefits decrease, and environmental investors aggressively tilt their portfolios to contribute more to the public good.

[15] Note that the initial wealth of each agent is set equal to 1, so in the extreme case where (i) all wealth is invested in the clean firm, and (ii) the clean firm is perfectly efficient in producing public goods ($\gamma_C = 0$), the total public good provision would be equal to 2.

5.2 Taxation and Crowding Out

Up to this point, we have assumed that the public good is exclusively provided by companies. Of course, the government also often provides public goods. An important result in the public economics literature is that public good provision by individuals is crowded out by government taxation/expenditure one-for-one, i.e., an additional dollar spent by the government reduces total contributions by one dollar (Bergstrom, Blume, and Varian (1986)). We now allow the government to play a role in public goods provision and study how government funding crowds out public good provision by companies funded by investors.

When the government levies tax τ from each investor, there is 2τ available to contribute towards G. To capture the inefficiency or any organizational costs in the government (Bandiera et al., 2009), we model a waste cost of λ, namely, for τ amount of taxes levied, only $(1-\lambda)\tau$ effectively contribute to public good provision.[16]

With taxes, the total amount of public good provision amounts to:

$$G(\tau) = 2\Big(\tau(1-\lambda) + (\omega_0 - \tau)\theta_C(\tau)(1-\gamma_C) + (\omega_0 - \tau)\theta_D(\tau)(1-\gamma_D)\Big). \quad (21)$$

Since investors take taxes as given and the tax is lump-sum, in this environment the only way taxes could affect the investors' portfolio choices is through the public good G (recall from Proposition 1 that $f'(G)$ affects investors' portfolio choices).[17]

We first focus on the case when $f''(G) = 0$ to show the key trade-off concerning crowd-out analytically. We will subsequently look at the case where $f''(G) < 0$. If $f''(G) = 0$, there is no wealth effect coming from G,

[16] Note that papers such as Bergstrom, Blume, and Varian (1986) do not model this distortion. However, it will be simple for us to compare to those models as well by setting $\lambda = 0$.

[17] In the classic Merton models of portfolio choice, the optimal equity share is independent of private wealth, so reduced wealth alone does not alter investors' choices. Since we modify the Merton (1987) utility function by incorporating a public good, we reintroduce the wealth effects through $f(G)$.

and so taxes should not affect portfolio choices. Lemma 2 summarizes and proves this observation.

Lemma 2. *When $f''(G) = 0$, levying τ from investors' initial wealth does not alter asset prices nor allocations.*

Given Lemma 2, Equation (21) no longer has allocations θ_C and θ_D depending on τ. We observe that the total quantity of public goods without taxes is $G = 2\Big(\omega_0\theta_C(1-\gamma_C) + \omega_0\theta_D(1-\gamma_D)\Big)$. This implies that:

$$G(\tau) - G = 2\tau\Big(-\lambda + \gamma_D - \theta_C(\gamma_D - \gamma_C)\Big). \tag{22}$$

Equation (22) is independent of investor wealth, and $\frac{G(\tau)-G}{\tau}$ is independent of taxation. This confirms that levying taxes only exerts a substitution effect, but no wealth effect. The substitution effect means that taxes substitute the public good away from the private sector to the government. From Equation (22), it follows that $G(\tau) - G > 0$ *iff* $\lambda < \gamma_D - (\gamma_D - \gamma_C)\theta_C$, which leads to Proposition 6.

Proposition 6. *Suppose $f''(G) = 0$. When the government levies taxes τ from each investor, it crowds out private provision of public goods by $2\tau(\lambda + \sum_n (1-\gamma_n)\theta_n)$. Therefore:*

1. *If $\lambda \leq \gamma_C$, the total provision of public goods always increases; hence, crowding out is not complete.*
2. *If $\lambda > \gamma_C$, whether total provision of public goods increases depends on the relative strength between private wealth allocation to the clean firm and the government's waste cost.*
   - *(a) If $\theta_C < \frac{\gamma_D - \lambda}{\gamma_D - \gamma_C}$, crowding out is not complete - the total provision of public goods increases.*
   - *(b) If $\theta_C > \frac{\gamma_D - \lambda}{\gamma_D - \gamma_C}$, the total provision of public goods decreases.*

The economic meaning of λ and γ_C can help us interpret the above results. The parameter λ reflects the inefficiency from levying taxes. The term γ_C is the production weight for clean firms - the higher γ_C is, the less efficient the clean firm is in producing public goods. When $\lambda \leq \gamma_C$, it means the government is sufficiently efficient or, on the flip side, the clean firm is sufficiently inefficient at public goods production, and thus, the total provision of public good always increases and there is not 100% crowding out. Interestingly, this result is independent of investors' portfolio choices.

When $\lambda \geq \gamma_C$, however, the amount of crowding out also depends on investors' portfolio choices. If the overall allocation to clean firms is sufficiently small (which might reflect that investors do not feel strongly about the environment), then even if clean firms are extremely efficient in producing public goods and government is inefficient, taxation nevertheless increases the total quantity of public goods and crowding out is not 100%. If investors care a lot about climate change mitigation and investment in clean firms is sufficiently large, government provision of public goods leads to excessive crowd out (more than 100%) - the total amount of public goods provided decreases. This possibility holds even though, in our model, the private provision of public goods has to go through a production process with diminishing returns. The intuition is that if there are large investments in clean firms already, then the economy is benefiting greatly from the efficient production of G by the clean firm. Taxation will hurt the total production of G given that λ is high.

We now show that the above trade-off is still present when $f''(G) < 0$. In the proof of Lemma 2, we have demonstrated that when $f''(G) < 0$, asset prices and allocations are both affected due to the indirect general equilibrium effect (wealth effect) of public goods affecting investors' portfolio choices. Let $\Delta\theta_n(\tau)$ be the difference between the market portfolio allocation to firm n with taxes and without taxes, i.e., $\Delta\theta_n(\tau) = \theta_n(\tau) - \theta_n$. The difference between public good provision with and without taxes can thus be expressed as:

$$G(\tau)-G = 2\Big(\tau(1-\lambda)+\omega_0\sum_n((1-\gamma_n)\Delta\theta_n(\tau))-\tau(\sum_n\theta_n(\tau)(1-\gamma_n))\Big), \quad (23)$$

Given that $\sum_n \theta_n(\tau) = \sum_n \theta_n = 1$, it follows that $\sum_n \Delta\theta_n(\tau) = 0$. We also know that $\sum_n \theta_n(\tau) = 1$. Then the above equation can be rewritten as

$$G(\tau) - G = 2\tau\Big(\underbrace{-\lambda + \gamma_D - \theta_C(\gamma_D - \gamma_C)}_{\text{substitution effect}} + \underbrace{(\omega_0/\tau - 1)(\gamma_D - \gamma_C)\Delta\theta_C(\tau)}_{\text{wealth effect}}\Big). \tag{24}$$

Comparing Equation (24) with Equation (22), the only difference is that $f''(G)$ introduces a wealth effect. This wealth effect may counteract the substitution effect. The above analytics lead to the following corollary.

Corollary 2. *If $f''(G) < 0$, given γ_n, whether total provision of public goods increases depends on the relative strength between private wealth allocation to the clean firm, the government's waste cost, and the tax rate. Formally, $G(\tau) - G > 0$ iff*

$$\lambda < \gamma_D - (\gamma_D - \gamma_C)\theta_C + \underbrace{(\omega_0/\tau - 1)(\gamma_D - \gamma_C)\Delta\theta_C(\tau)}_{\textit{due to wealth effect}} \tag{25}$$

The only change in the conditions between Corollary 2 and Proposition 6 is due to the wealth effect. If condition (25) holds, total public goods provision increases. We make two remarks here. First, the wealth effect $(\omega_0/\tau - 1)(\gamma_D - \gamma_C)\Delta\theta_C(\tau)$ should be negative. This is because as the total quantity of public goods increases due to taxes, $f'(G(\tau)) - f'(G) < 0$, the marginal utility of public goods for the investors decreases, so in the aggregate, investors will allocate less wealth to the clean firms, implying that $\Delta\theta_C(\tau) < 0$. This means the wealth effect will put a downward pressure on the right-hand side of Equation (25). Therefore, with the wealth effect present, the condition for the total public goods to increase will be stricter than the case with no wealth effect ($f''(G) = 0$).

We illustrate the results for the model where $f''(G) < 0$ in Figures 5 and 6. In Figure 5 in the left panel, we use the benchmark value of γ_C (the output elasticity of capital for Firm C) and vary λ. While it is possible when levels of λ are high for public good provision with taxation to dip below the provision in the model without taxation, these high levels (approximately above 0.7) seem unrealistic. In the middle panel, we use a much lower value for the output elasticity of capital for the clean firm, setting $\gamma_C = 0.2$. This makes a big difference, showing a reduction in public good provision under taxation for values approximately above $\lambda = 0.3$. This is clearly being driven by the fact that Firm C has become a very efficient producer of the public good and its contribution is being crowded out. Of course, this is for the non-standard situation where the clean firm is very efficient at producing the public good (and not very efficient at producing the private good).

In Figure 6, we examine how public good provision changes for different levels of the tax rate τ. In the left panel, we see that for our benchmark parameters in which $\lambda = 0.2$ (see Table 1 for further discussion), the imposition of taxes increases the provision of the public good. Interestingly, the slope of the line is approximately 0.87, implying that while there is not complete crowding out, the taxes raised do not increase the public good provision one-for-one. In the middle panel, we use a higher value for the waste cost, setting $\lambda = 0.4$, and the slope of the line is much flatter. In the right panel, as we use an extremely high value for λ of 0.8, the provision of the public good is below the amount when there are no taxes, indicating excessive crowd out. Moreover, as taxes increase, the crowd out gets more excessive.[18]

Our results provide one explanation for the surge in sustainable investment right after Donald Trump's surprise 2016 election. Donald Trump's platform against climate change mitigation and his threat to pull out of the Paris accord (which he did) implied a substantial decrease in governmental support for the environment - based on expected cash flows alone, climate friendly stocks should have gone down. Nevertheless, there was a surge in sustainable investment[19] and Ramelli et al. (2021) find that firms with a high level of climate responsibility had a high abnormal return right after the

[18]In simulations not included in this paper, we get the same pattern (from "partial crowd out" to "excessive crowd out") by increasing the efficiency of the clean firm in producing the public good, or increasing the preferences for the public good, while keeping the waste cost at the benchmark value of 0.2.

[19]In the mutual fund space, inflows into ESG mutual funds in the first half of 2017 were \$3.5 billion (compared with \$4.9 billion for the whole of 2016). "The Trump White House gave these mutual funds a big boost," *Fortune.com*, June 16, 2018.

election. Our results suggest the expected decrease in government funding could have prompted investors to react by compensating for the shortfall. Therefore government inaction might, in part, explain the rise of sustainable finance.

Of course, our results point out that this may or may not have been a 'silver lining' in reaction to Trump's policies. Sustainable investing may have increased, but this may have resulted in either higher *or* lower provision of the public good relative to before Trump's election.

5.3 Green Subsidies

Our model provides a natural laboratory to study the efficacy of a green subsidy, which has become a commonly used policy tool. For example, the U.S. Inflation Reduction Act allocates \$5.8 billion directly to and expands tax credits to manufacturers who decarbonize; in addition, it allocates a substantial amount more to subsidize clean energy production and lower clean energy prices.[20] We consider a green subsidy where a firm receives δz dollars of subsidy for the green investment z a firm makes, where $\delta \in (0, 1)$ (the setup and proofs are in Appendix A.12). Let $\bar{r}_n(\delta)$ be the expected asset return with the subsidy and $\bar{r}_n$ be the return without the subsidy. We denote $z(\delta)$ as the total green investment with the subsidy. As in the previous section, we allow for a waste cost of λ, i.e., for \$1 of taxes raised, only \$1$-$ λ is available for the subsidy.

We find that the equilibrium has the following properties:

Proposition 7. *In equilibrium,*

1. *Firm n's green investment* z_n *is:*

$$z_n = \frac{1 - \gamma_n}{1 - \delta} k_n. \tag{26}$$

2. *For any level of green subsidy* $\delta \in (0, 1)$*,* $\bar{r}_n(\delta) > \bar{r}_n$*, and* $\frac{d\left(\frac{1+\bar{r}_n(\delta)}{1+\bar{r}_n}\right)}{d\gamma_n} <$

[20]See details in "Building a clean energy economy: A guidebook to the inflation reduction act's investments in clean energy and climate action." by The White House, January 2023, Version 2.

0. *A green subsidy does not change the asset pricing implications of condition* (14) *for CAPM alphas.*

3. *The total amount of public good provided has a non-linear relationship with the green subsidy, and*

$$G = \frac{\omega_0}{\Xi + \Phi},$$

where $\Xi \equiv \frac{1-\delta}{2(1-\gamma_D+(\gamma_D-\gamma_C)\theta_C)}$ *and* $\Phi \equiv \frac{\delta}{2(1-\lambda)}$*. Note that* $\frac{d\Phi}{d\delta} > 0$ *and whenever the semi-elasticity* $\epsilon_{\theta_C,\delta} > (\delta - 1)^{-1}$, $\frac{d\Xi}{d\delta} < 0$*.*

The above proposition says that given firm capital k_n, the higher the subsidy is, the higher the green investment is. A firm's expected excess return is larger than the case with no subsidy, and this difference decreases with firm's carbon intensity γ_n. Nevertheless, we prove in the appendix that the asset pricing implications given by condition (14) still hold. Furthermore, we uncover a non-linear relationship between the total amount of public good provided and the subsidy. This is due to a trade-off. On the one hand, the green subsidy is corrective and aims to mitigate free-riding and encourage green investment. It effectively reduces the cost of green investment and nudges the firm to substitute away from traditional investment (Equation (26)), so there is a positive impact of δ on G when $\frac{d\Xi}{d\delta} < 0$. On the other hand, the subsidy comes from taxing private wealth and, thus, reduces the total amount of productive capital, which shifts inwards the production possibility frontier. This is a direct negative impact of the subsidy δ on G, and when the waste parameter λ increases, the direct negative impact increases, i.e., $\frac{d^2\Phi}{d\delta d\lambda} > 0$.

To examine the allocation and social welfare implications with heterogeneous firms, we solve the model numerically (see Figure 7). Social welfare (top left panel) is non-monotonic in the green subsidy. It first increases with δ and then decreases; when the green subsidy is extremely high (close to 0.5 in our benchmark parameter space), the social welfare drops below the baseline equilibrium without the green subsidy. This is because large green subsidies crowd out the private good production (bottom left panel of Figure

7). Interestingly, when the clean firm's green technology is sufficiently superior (setting $\gamma_C = 0.2$), the green subsidy does not always encourage public goods provision. As illustrated in the bottom right panel of Figure 7, below a threshold, more green subsidy leads to more public goods provision, and above this threshold, more green subsidy decreases public goods provision. This is because the negative impact of subsidy on the overall production possibility frontier starts to outweigh its positive impact on the public good.

Finally, we compare the effectiveness of the green subsidy to taxes (analyzed in Section 5.2). We solve numerically for the optimal green subsidy, which we define as the subsidy that maximizes the sum of investor utilities. We demonstrate in Figure 8 that, for a given set of parameters, the optimal green subsidy is more effective than the optimal tax. For a wide range of investor preference ψ_E, the optimal tax is zero, so that there is no difference between the maximum utility in the base case and with taxes, while the optimal subsidy δ is between 0.1 and 0.3. The optimal tax starts diverging from zero only for ψ_E around 2.7, but the corresponding social welfare remains lower than what can be obtained with the optimal green subsidy for the same level of investor's preference for the public good. Interestingly, in unreported results we show that this is true even when we set the government waste cost λ to zero.[21] This result suggests that the private provision of public goods using corrective policy instruments such as subsidies can be more effective than taxes. This may be because the subsidy affects both the public good provision and private good production on the margin, whereas taxes directly take a lump-sum block of resources away from the private goods production.

6 Extensions

We return to the main model and extend it in three directions: introducing negative externalities, allowing for investors to donate some of their wealth

[21] The reason is similar to the private donation case in Section 6.2 where we show that for a wide range of investor preference ψ_E, even though donations are permitted, they are not used by each investor.

directly to the public good, and making the provision of the public good uncertain.

6.1 Negative Externalities

In this section, we extend the model to consider negative externalities. In contrast to the main model, where the dirty firm contributes less positively to the public good than the clean firm, here the dirty firm's investment z_D contributes *negatively* to the public good, i.e., $G = z_C - z_D$.[22]

Since the dirty firm does not contribute to G positively, we no longer need to assume $\gamma_C < \gamma_D$. When $\gamma_C > \gamma_D$, it means the dirty firm's marginal productivity from polluting is higher than the clean firm's marginal productivity from mitigating climate change, and vice versa. We solve for the equilibrium value of η_D, the marginal contribution to the public good from the portfolio allocation to the dirty firm, which is $\eta_D = \gamma_D - 1$. Since $\gamma_D < 1$, η_D is negative. The following proposition summarizes the portfolio allocation and asset pricing properties, and the proof is in Appendix A.13.

Proposition 8. *With a negative externality, in equilibrium (and focusing on the case when $\bar{r}_M > 0$):*

1. *Investor E invests more in the clean firm and less in the dirty firm than investor F.*

2. *Whenever $\gamma_D < \gamma_C$ ($\gamma_D > \gamma_C$), investor E invests less (more) in the stock market than investor F.*

3. *The clean firm exhibits a negative alpha iff $\frac{1-\gamma_C}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} > \beta_C$ (and positive alpha iff $\frac{1-\gamma_C}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} < \beta_C$); the dirty firm exhibits a positive alpha iff $\frac{\gamma_D-1}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} < \beta_D$ (and negative alpha iff $\frac{\gamma_D-1}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} > \beta_D$).*

The negative externality changes the portfolio allocation and asset pricing implications compared to the results of our main model. Whereas our

[22] This formulation is similar to that in the dynamic macro literature on growth and taxation with carbon externalities (Acemoglu et al. (2012), Barrage (2020)).

main model demonstrates the possibility of investor E investing more in the dirty firm than investor F, in the negative externality case, investor E invests less in the dirty firm than investor F unambiguously. Similarly, in the main model investor E always invests more in the stock market than investor F, but in the negative externality case, she may or may not invest more in the stock market than investor F. This is because the diversification benefits of trading dirty stocks come with the cost of reducing investor E's utility for the public good (more than for investor F). In terms of asset pricing, when the market expected excess return is positive, the CAPM-alpha is ambiguous (as is in the main model) because it depends on the firm's systematic risk and its relative public good contribution or damage.

We provide a simulation of the model, solving for the amount of public good provided, the asset allocation and the alpha, while varying exogenous parameters (see Figures 9, 10, and 11). One might expect the total public good provision to be always lower in the negative externality case than the main model; after all, the dirty firm's production in this negative externality case reduces the public good. However, by comparing Figure 9 and Figure 1, we find that when ψ_E or ψ_F is sufficiently high, or risk aversion is sufficiently low, public good provision in the negative externality case is larger than in the main model. This is because the market allocation to the dirty firm in these parameter ranges is close to zero in the negative externality case; whereas the market allocation to the dirty firm is positive in the main model. This implies that in these parameter ranges, the total amount of capital allocated to the clean firm is higher in the negative externality case than the main model, and, thus, the total public good provision is higher.

Figure 11 shows the firm alphas of the negative externality case. Compared with Figure 3, Figure 11 shows a similar profile of firm alphas. The clean firm tends to exhibit a negative alpha, and the dirty firm tends to have a positive alpha. However, we can see that the dirty firm's positive alpha in the negative externality case tends to be much higher than the main model. This is due to the lower allocations to the dirty firm (see Figure 10), which lower its market beta.

6.2 Donations

In this section, we assume investors also have the possibility to directly donate their wealth to the public good in addition to investing in financial markets. This can be interpreted as investors contributing to charities or NGOs that provide the public good; this represents a different route for public good provision than financial markets.[23]

Let d_i be investor i's donation to the public good, where $d_i \geq 0$. We use ϕ_i to denote the Lagrangian multiplier of the constraint $d_i \geq 0$. Let $\bar{R}_i$ denote investor i's expected gross portfolio return, i.e., $\bar{R}_i = 1 + r_f + \sum_n \bar{r}_n \theta_{i,n}$. Let σ_i^2 denote the variance of investor i's portfolio. Although we assumed that taxation incurs a waste cost λ in the previous section, for simplicity we assume there is no inefficiency *per se* associated with donations.[24]

Lemma 3 below summarizes the key trade-off for making donations.

Lemma 3. *The optimality condition for investor i's donation choice is:*

$$\bar{R}_i = \frac{\nu}{2}\sigma_i^2 + \psi_i f'(G)\Big(1 - \sum_n (1-\gamma_n)\theta_{i,n}\Big) + \phi_i. \tag{27}$$

The marginal cost of a donation is the financial return of investing in the market that the investor gives up, i.e., the left-hand side of (27). The marginal benefit of a donation consists of (i) a reduction in the stock market risks in her portfolio ($\frac{\nu}{2}\sigma_i^2$) and (ii) the marginal contribution to the public good, i.e., $\psi_i f'(G)(1-\sum_n(1-\gamma_n)\theta_{i,n})$.[25] If the Lagrangian multiplier $\phi_i = 0$, investor i chooses the level of donation i such that the marginal benefit equals the marginal cost. If $\phi_i > 0$, the marginal cost is larger than the marginal benefit, so she simply does not donate. Lemma 3 demonstrates that investors may choose not to donate when (i) their expected portfolio return is sufficiently high, (ii) portfolio risk is sufficiently low, and/or (iii) the amount of public good is already sufficiently high. Therefore one may expect

[23] We shut down the government provision explored in Sections 5.2 and 5.3.

[24] In reality, there may be agency costs or inefficiencies in selecting the right organizations for public goods provision.

[25] Note that the marginal contribution to the public good is the value of the donation minus the reduced provision due to lower financial investment.

that there are empirical linkages between financial market performance and philanthropy.

Donations affect asset prices and allocations exactly the same way as taxes in Section 5.2. When $f''(G) = 0$, donations do not alter asset prices nor allocations; when $f''(G) < 0$, asset prices and allocations are affected through the indirect general equilibrium effect (wealth effect) of the public good affecting investors' portfolio choices. Our key results on asset pricing in Section 3 all go through in both cases.

We take advantage of the fact that donations do not alter asset prices nor allocations when $f''(G) = 0$ to show that the total amount of public good when allowing for donations is larger than when donations are not allowed:

Proposition 9. *When $f''(G) = 0$ and $\gamma_C > \frac{1}{2}$, the amount of public good allowing for donations is weakly larger than the case where donations are not allowed.*

We note that this proof relies on assuming that $\gamma_C > \frac{1}{2}$. We also use this assumption in the simulations, and it is quite natural: it means that all firms (both clean and dirty) are more tilted toward producing private goods than public ones.

The proposition states that the amount of public goods is weakly larger when donations are allowed. In the proof we show that the amount is strictly larger when at least one investor donates a positive amount.

We examine the case where $f''(G) < 0$ in the simulation. In Figure 12, in the third panel, we show that the amount of public good allowing for donations is weakly larger than when donations are not allowed. For a wide range of ψ_E, even though donations are permitted, they are not used by either investor. Once ψ_E becomes sufficiently large (approximately $\psi_E = 3.7$ in the simulation), the environmentally conscious investor begins to donate. Her donations increase sharply from this point and can be seen in the first panel. The middle panel of Figure 12 shows that the increase in donations from the environmentally conscious investor implies that she invests in the clean firm less aggressively. The financial investor also pivots to investing more in the dirty firm, which represents free riding.

6.3 Risky Public Good

Particularly in the case of carbon emissions, there is substantial uncertainty around whether benefits will materialize from reductions in emissions (e.g., Nordhaus, 2014). In our model, this would be the risk that the public good does not provide as many benefits as expected. We model this risk (see Appendix A.16) by assuming that there is a noise term that may shock public good production with mean 1 and variance σ_G^2. We also assume that it may interact with the other risks in the model, i.e., the variance in the productivity of the clean and dirty firms (represented by ρ_{CG} and ρ_{DG}, respectively).

The model behaves in the same way as the main model but there are two things specific to this model to note. First, as the correlation between the clean firm's productivity and the public good provision increases, the environmental investor invests less in the clean firm (shown in the left graph of Figure 13). An investor who cares more about the environment invests less in it because it exposes her to a correlated risk of climate change measures failing and her investments failing at the same time. Second, as the risk of public good provision goes up, the expected amount of public good provided goes down (shown in the right graph of Figure 13). This shows that if the climate mitigation measures are more risky, fewer of them are produced.

7 Conclusion

We construct an asset pricing model with production and public good provision in order to explore the role of sustainable investment. We find that environmentally conscious investors are less risk averse and invest more due to their added utility from public good consumption. They may also invest more in dirty firms to hedge themselves. Clean firms may have positive or negative alpha relative to a standard CAPM benchmark; they will have positive alpha when they hold a lot of systematic risk. The public good is underprovided and government funding can crowd out investing. A green subsidy may dominate a tax to fund the public good.

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A Appendix

A.1 Firm Maximization

First, take the FOC for k_n:

$$\mathbb{E}_0(\tilde{M}\tilde{\epsilon}_n)\gamma_n(k_n - z_n)^{\gamma_n - 1} z_n^{1-\gamma_n} = 1, \tag{28}$$

Next, take the FOC for z_n:

$$-\gamma_n\mathbb{E}_0(\tilde{M}\tilde{\epsilon}_n)(k_n - z_n)^{\gamma_n - 1} z_n^{1-\gamma_n} + (1-\gamma_n)\mathbb{E}_0(\tilde{M}\tilde{\epsilon}_n)(k_n - z_n)^{\gamma_n} z_n^{1-\gamma_n} = 0. \tag{29}$$

The FOC (28) can be rearranged as $\mathbb{E}_0(\tilde{M}\tilde{\epsilon}_n) = \gamma_n^{-1}(k_n - z_n)^{1-\gamma_n} z_n^{\gamma_n - 1}$, and substituted into (29) to obtain:

$$\frac{k_n - z_n}{z_n} = \frac{\gamma_n}{1-\gamma_n}, \tag{30}$$

Which is equivalent to:

$$z_n = (1-\gamma_n)k_n. \tag{31}$$

A.2 Proof of Proposition 1 (Asset allocation)

Investor i ($i \in (E, F)$)'s utility function can be rewritten as:

$$\mathbb{E}_0(\tilde{\omega}_i) - \nu Var(\tilde{\omega}_i)/(2\omega_0) + \psi_i f(G)$$

where:

$$E_0\tilde{\omega}_i = \omega_0(1 + r_f + \bar{r}_C\theta_{i,C} + \bar{r}_D\theta_{i,D}), \tag{32}$$

$$Var\tilde{\omega}_i = \omega_0^2\sigma^2\theta_{i,C}^2 + \omega_0^2\sigma^2\theta_{i,D}^2 + 2\theta_{i,C}\theta_{i,D}\omega_0^2\sigma^2\rho. \tag{33}$$

The FOCs with respect to $\theta_{i,C}$ and $\theta_{i,D}$ are:

$$\bar{r}_C + (1-\gamma_C)\psi_i f'(G) = \nu\sigma^2\theta_{i,C} + \nu\sigma^2\rho\theta_{i,D}, \tag{34}$$

$$\bar{r}_D + (1-\gamma_D)\psi_i f'(G) = \nu\sigma^2\theta_{i,D} + \nu\sigma^2\rho\theta_{i,C}. \tag{35}$$

Solving for $\theta_{i,C}$ and $\theta_{i,D}$:

$$\theta_{i,C} = \frac{\Big(1-\gamma_C-\rho(1-\gamma_D)\Big)\psi_i f'(G) + \bar{r}_C - \bar{r}_D\rho}{\nu(1-\rho^2)\sigma^2}, \tag{36}$$

$$\theta_{i,D} = \frac{\Big(1-\gamma_D-\rho(1-\gamma_C)\Big)\psi_i f'(G) + \bar{r}_D - \bar{r}_C\rho}{\nu(1-\rho^2)\sigma^2}. \tag{37}$$

Define $\psi = \frac{\psi_C+\psi_D}{2}$. We combine investors' FOCs for asset allocations to get:

$$\bar{r}_C + (1-\gamma_C)\psi f'(G) = \nu\sigma^2(\sum_{i\in(E,F)}\theta_{i,C}/2 + \rho\sum_{i\in(E,F)}\theta_{i,D}/2), \tag{38}$$

$$\bar{r}_D + (1-\gamma_D)\psi f'(G) = \nu\sigma^2(\sum_{i\in(E,F)}\theta_{i,D}/2 + \rho\sum_{i\in(E,F)}\theta_{i,C}/2). \tag{39}$$

The above two equations in matrix form are equivalent to:

$$\bar{r} = \nu\Sigma\Theta - (1-\gamma)\psi f'(G). \tag{40}$$

□

A.3 Equivalence of Stock Price Formulation

The model formulates investors' maximization in terms of excess returns. Here we show that a formulation using stock prices is equivalent. Suppose each firm issues 1 share. Let $q_{i,n}\omega_0$ be the quantity of firm n shares that investor i purchases, and p_n be the price of a share of firm n at $t=0$. Let $q_{i,f}\omega_0$ be the quantity of riskless asset investor i trades, and p_f be the price of the riskless asset.

- The payoff of the riskless asset at $t = 1$ is 1, so the risk-free rate is simply $1 + r_f = \frac{1}{p_f}$.

- The payoff of firm $n(n \in (C, D))$'s stock at $t = 1$ is $\tilde{y}_n$, so its return is simply $1 + r_n = \frac{\tilde{y}_n}{p_n}$. Since $\tilde{y}_n = \tilde{A}_n k_n$, and market clearing for capital gives

$$k_n = p_n(\sum_{i \in (E,F)} q_{i,n}\omega_0). \tag{41}$$

As firm n issues 1 share,

$$\sum_{i \in (E,F)} q_{i,n}\omega_0 = 1. \tag{42}$$

Combining (41) and (42), we get

$$\tilde{y}_n = \tilde{A}_n p_n, \tag{43}$$

and it follows that $1 + r_n = \tilde{A}_n$.

- Excess return is therefore $\tilde{r}_n = \tilde{A}_n - \frac{1}{p_f}$. This means $1 + r_f + \tilde{r}_n = \tilde{A}_n$, consistent with what we already have in the manuscript.

Let us observe investor i's budget constraints.

The budget constraint at date $t = 0$ is:

$$\sum_{n \in (C,D)} p_n q_{i,n}\omega_0 + p_f q_{i,f}\omega_0 = \omega_0. \tag{44}$$

The budget constraint at date $t = 1$ is:

$$\tilde{\omega}_i = \sum_{n \in \{C,D\}} \tilde{y}_n q_{i,n}\omega_0 + q_{i,f}\omega_0. \tag{45}$$

Combining the budget constraints, it follows that:

$$\tilde{\omega}_i = \Big((\tilde{A}_C - \frac{1}{p_f})p_C q_{iC} + (\tilde{A}_D - \frac{1}{p_f})p_D q_{iD} + \frac{1}{p_f}\Big)\omega_0. \tag{46}$$

Also by definition:

$$p_n q_{i,n}\omega_0 = \theta_{i,n}\omega_0. \tag{47}$$

We have shown $\tilde{r}_n = \tilde{A}_n - \frac{1}{p_f}$, and by definition $1 + r_f = \frac{1}{p_f}$. And from Equations (46) and (47), we have

$$\tilde{\omega}_i = \omega_0(1 + r_f + \sum_{n\in(i,D)} \tilde{r}_n\theta_{i,n}). \tag{48}$$

Furthermore, from Equation (43) and $\tilde{y}_n = \tilde{A}_n k_n$, we have $p_n = \frac{\tilde{y}_n}{\tilde{A}_n} = k_n$. Higher capital allocation to the firm leads to a higher equity price. Since we have CRS and firms issue 1 unit of stock, the stock price is simply equal to the capital allocation in equilibrium. Finally, to solve for price p_n, we use Equations (47), (42) (market clearing), and Θ_i (representations of demand) from Appendix A.2.

A.4 Proof of Corollary 1 (Allocation, continued)

Investor i's wealth in the stock market amounts to:

$$\omega_0((2 - \sum_n \gamma_n)(1-\rho)\psi_i f'(G) + \sum_n \bar{r}_n(1-\rho))(\nu(1-\rho^2)\sigma^2)^{-1}.$$

It follows that investor E invests more than investor F because $\psi_E > \psi_F$. Because two investors have the same initial wealth, their allocations to the risk-free asset $\theta_{E,f}$ and $\theta_{F,f}$ must satisfy $\theta_{E,f} < 0 < \theta_{F,f}$ for the $t = 0$ budget constraints to hold.

Since $f'(G) > 0$ and $\psi_E > \psi_F$, from Equation (36) derived in Proposition A.2, we can see that if $1-\gamma_D-\rho(1-\gamma_C) < 0$, which is equivalent to $\rho > \frac{1-\gamma_D}{1-\gamma_C}$, then $\theta_{E,D} < \theta_{F,D}$, and vice versa. □

A.5 Proof of Lemma 1 (Risk Aversion)

Measurement of risk aversion in our setting is complicated by the presence of multiple goods, i.e. the private and public goods. To compute risk aversion in these cases, Stiglitz (1969) proposed using the indirect utility function and reformulating the problem from measuring risk aversion with respect to multiple goods to measuring risk aversion with respect to a single good, wealth. In the dynamic setting, Constantinides (1990) proposed measuring risk aversion using the household's value function, which again collapses the problem of measuring risk aversion with respect to an infinity of goods across time and states of nature into measuring risk aversion with respect to a single good, initial household wealth.

The absolute risk aversion is given by the negative of the ratio of the second and first derivatives of the value function, with respect to wealth.

For investor E, the utility function is given by:

$$U_E = \mathbb{E}_0(\tilde{\omega}_E) - \nu Var(\tilde{\omega}_E)/(2\omega_0) + \psi_E f(G),$$

where $G = H(\theta_{E,n}, \theta_{F,n})$ and ω_E denotes her stochastic terminal wealth, which is given by:

$$\tilde{\omega}_E = \omega_0(1 + r_f + \tilde{r}_C\theta_{E,C} + \tilde{r}_D\theta_{E,D}) = v(\theta_{E,C}, \theta_{E,D}).$$

Agent E's value function is therefore given by (see also Equations (32) and (33)):

$$\begin{aligned} V_E &= \omega_0(1 + r_f + \bar{r}_C\theta_{E,C} + \bar{r}_D\theta_{E,D}) - \frac{\nu}{2\omega_0}[\omega_0^2\sigma^2\theta_{E,C}^2 + \omega_0^2\sigma^2\theta_{E,D}^2 + 2\theta_{E,C}\theta_{E,D}\omega_0^2\sigma^2\rho] \\ &\quad +\psi_E f(H(\theta_{E,n}, \theta_{F,n})). \end{aligned}$$

We can now compute partial derivatives of V_E with respect to $\theta_{E,C}$ and

$\theta_{E,D}$.

$$\frac{\partial V_E}{\partial \theta_{E,C}} = \omega_0 \bar{r}_C - \frac{\nu}{2\omega_0}[2\omega_0^2 \sigma^2 \theta_{E,C} + 2\theta_{E,D}\omega_0^2\sigma^2\rho] + \psi_E f'(G)\frac{\partial H}{\partial \theta_{E,C}}$$
$$= \omega_0[\bar{r}_C - \nu\sigma^2\theta_{E,C} + \nu\sigma^2\rho\theta_{E,D} + f'(G)\psi_E(1-\gamma_C)] \quad (49)$$
$$\frac{\partial V_E}{\partial \theta_{E,D}} = \omega_0 \bar{r}_D - \frac{\nu}{2\omega_0}[2\omega_0^2 \sigma^2 \theta_{E,D} + 2\theta_{E,C}\omega_0^2\sigma^2\rho] + \psi_E f'(G)\frac{\partial H}{\partial \theta_{E,D}}$$
$$= \omega_0[\bar{r}_D - \nu\sigma^2\theta_{E,D} + \nu\sigma^2\rho\theta_{E,C} + f'(G)\psi_E(1-\gamma_D)] \quad (50)$$
$$\frac{\partial^2 V_E}{\partial \theta_{E,C}^2} = -\omega_0\nu\sigma^2 = \frac{\partial^2 V_E}{\partial \theta_{E,D}^2} \quad (51)$$
$$\frac{\partial^2 V_E}{\partial \theta_{E,C}\partial \theta_{E,D}} = \omega_0\nu\rho\sigma^2. \quad (52)$$

The full derivatives of V_E are therefore:

$$dV_E = \frac{\partial V_E}{\partial \theta_{E,C}}d\theta_{E,C} + \frac{\partial V_E}{\partial \theta_{E,D}}d\theta_{E,D}$$
$$= \omega_0[\bar{r}_C - \nu\sigma^2\theta_{E,C} + \nu\sigma^2\rho\theta_{E,D} + f'(G)\psi_E(1-\gamma_C)]d\theta_{E,C}$$
$$+\omega_0[\bar{r}_D - \nu\sigma^2\theta_{E,D} + \nu\sigma^2\rho\theta_{E,C} + f'(G)\psi_E(1-\gamma_D)]d\theta_{E,D} \quad (53)$$

and
$$dV_E^2 = \frac{\partial^2 V_E}{\partial \theta_{E,C}^2}d\theta_{E,C}^2 + \frac{\partial^2 V_E}{\partial \theta_{E,D}^2}d\theta_{E,D}^2 + 2\frac{\partial^2 V_E}{\partial \theta_{E,C}\partial \theta_{E,D}}d\theta_{E,C}d\theta_{E,D}$$
$$= -\omega_0\nu\sigma^2[d\theta_{E,C}d\theta_{E,C} + d\theta_{E,D}d\theta_{E,D} - 2\rho d\theta_{E,C}d\theta_{E,D}] \quad (54)$$

The absolute risk aversion for agent E is then:

$$ARA_E = \frac{\nu\sigma^2[d\theta_{E,C}d\theta_{E,C} + d\theta_{E,D}d\theta_{E,D} - 2\rho d\theta_{E,C}d\theta_{E,D}]}{[\bar{r}_C - \nu\sigma^2\theta_{E,C} + \nu\sigma^2\rho\theta_{E,D} + f'(G)\psi_E(1-\gamma_C)]d\theta_{E,C} + [\bar{r}_D - \nu\sigma^2\theta_{E,D} + \nu\sigma^2\rho\theta_{E,C} + f'(G)\psi_E(1-\gamma_D)]d\theta_{E,D}}$$

An equivalent expression holds for agent F and we know that $\psi_E > \psi_F$. Therefore we obtain $ARA_E < ARA_F$. In general, the presence of $\psi_i f(G)$ in the utility function reduces the curvature, i.e. the risk aversion, for the agent with the highest ψ_i. $\square$

A.6 Proof of Proposition 2 (Asset pricing)

Combining Equations (8) and (10) yields:

$$\bar{r} = \frac{\Sigma\Theta - \nu^{-1}(1-\gamma)\psi f'(G)}{\sigma_M^2 - \nu^{-1}(1-\sum_n \theta_n\gamma_n)\psi f'(G)}\bar{r}_M. \tag{55}$$

□

A.7 Proof of Proposition 3 (Determinants of alpha)

Since $0 < \gamma_C < \gamma_D < 1$, it follows that:

$$\frac{1-\gamma_C}{1-\sum_n \theta_n\gamma_n} > 1 > \frac{1-\gamma_D}{1-\sum_n \theta_n\gamma_n}. \tag{56}$$

Moreover, β_C can be re-expressed as $\beta_C = (\theta_C + \theta_D\rho)(\theta_C^2 + \theta_D^2 + 2\theta_C\theta_D\rho)^{-1}$, and β_D can be re-expressed as $\beta_D = (\theta_D + \theta_C\rho)(\theta_C^2 + \theta_D^2 + 2\theta_C\theta_D\rho)^{-1}$. If $\beta_C \leq 1$, then $\theta_C + \theta_D\rho \leq \theta_C^2 + \theta_D^2 + 2\theta_C\theta_D\rho$, and given that $\theta_D = 1 - \theta_C$, the previous condition is equivalent to:

$$(\theta_D - \theta_C)\theta_D(\rho - 1) \leq 0, \tag{57}$$

which means $\theta_C \leq \theta_D$ has to hold. We can also see that $\theta_C \leq \theta_D$ leads to $\beta_C \leq 1$.

Turning to β_D, similarly we can show that $\theta_C \leq \theta_D$ is a sufficient and necessary condition for $\beta_D \geq 1$.

Thus, if $\beta_C \leq 1$, it follows that $\theta_C \leq \theta_D$ and $\beta_D \geq 1$, and given (56), we obtain:

$$\beta_C < \frac{1-\gamma_C}{1-\sum_n \theta_n\gamma_n}, \tag{58}$$

and

$$\beta_D > \frac{1-\gamma_D}{1-\sum_n \theta_n\gamma_n}. \tag{59}$$

The above two conditions mean that the clean firm has a negative CAPM alpha, and the dirty firm has a positive CAPM alpha.

Let us now turn to the case $\beta_C \geq 1$. As above, we can show that the sufficient and necessary condition for $\beta_C \geq 1$ to hold is $\theta_C \geq \theta_D$. And in this case $\beta_D \geq 1$. Therefore, from (56), we cannot draw analytic comparisons between β_C and $\frac{1-\gamma_C}{1-\sum_n \theta_n \gamma_n}$, and between β_D and $\frac{1-\gamma_D}{1-\sum_n \theta_n \gamma_n}$; thus, the effect on alpha is ambiguous. $\square$

A.8 Proof of Proposition 4 (Planner's equilibrium)

The planner chooses $\theta^p_{i,n}$ to maximize the following social welfare function

$$\frac{1}{2}\mathbb{E}_0\tilde{\omega}_E + \frac{1}{2}\mathbb{E}_0\tilde{\omega}_F - \frac{\nu}{4\omega_0}\left[Var(\tilde{\omega}_E) + Var(\tilde{\omega}_F)\right] + \frac{1}{2}(\psi_E + \psi_F)f(G)$$

where:

$$E_0\tilde{\omega}_E = \omega_0(1 + r_f + \bar{r}^p_C\theta^p_{E,C} + \bar{r}^p_D\theta^p_{E,D}), \tag{60}$$

$$E_0\tilde{\omega}_F = \omega_0(1 + r_f + \bar{r}^p_C\theta^p_{F,C} + \bar{r}^p_D\theta^p_{F,D}), \tag{61}$$

$$Var(\tilde{\omega}_E) = \omega_0^2\sigma^2(\theta^p_{E,C})^2 + \omega_0^2\sigma^2(\theta_{E,D})^2 + 2\theta^p_{E,C}\theta^p_{E,D}\omega_0^2\sigma^2\rho, \tag{62}$$

$$Var(\tilde{\omega}_F) = \omega_0^2\sigma^2(\theta^p_{F,C})^2 + \omega_0^2\sigma^2(\theta^p_{F,D})^2 + 2\theta^p_{F,C}\theta^p_{F,D}\omega_0^2\sigma^2\rho. \tag{63}$$

Taking FOC for $\theta^p_{E,C}$ and $\theta^p_{E,D}$:

$$\bar{r}^p_C + (1-\gamma_C)(\psi_E + \psi_F)f'(G^p) = \nu\sigma^2\theta^p_{E,C} + \nu\sigma^2\rho\theta^p_{E,D}, \tag{64}$$

$$\bar{r}^p_D + (1-\gamma_D)(\psi_E + \psi_F)f'(G^p) = \nu\sigma^2\theta^p_{E,D} + \nu\sigma^2\rho\theta^p_{E,C}. \tag{65}$$

Rearranging for $\theta^p_{E,C}$ and $\theta^p_{E,D}$:

$$\theta^p_{E,C} = \frac{\Big(1 - \gamma_C - \rho(1-\gamma_D)\Big)\sum_i \psi_i f'(G^p) + \bar{r}^p_C - \bar{r}^p_D\rho}{\nu(1-\rho^2)\sigma^2}, \tag{66}$$

$$\theta^p_{E,D} = \frac{\Big(1 - \gamma_D - \rho(1-\gamma_C)\Big)\sum_i \psi_i f'(G^p) + \bar{r}^p_D - \bar{r}^p_C\rho}{\nu(1-\rho^2)\sigma^2}. \tag{67}$$

Taking the FOC for $\theta^p_{F,C}$ and $\theta^p_{F,D}$:

$$\bar{r}^p_C + (1-\gamma_C)(\psi_E + \psi_F)f'(G^p) = \nu\sigma^2\theta^p_{F,C} + \nu\sigma^2\rho\theta^p_{F,D}, \tag{68}$$

$$\bar{r}_D + (1-\gamma_D)(\psi_E + \psi_F)f'(G^p) = \nu\sigma^2\theta^p_{F,D} + \nu\sigma^2\rho\theta^p_{F,C}. \tag{69}$$

Rearranging for $\theta^p_{F,C}$ and $\theta^p_{F,D}$:

$$\theta^p_{F,C} = \frac{\Big(1 - \gamma_C - \rho(1-\gamma_D)\Big)\sum_i \psi_i f'(G^p) + \bar{r}^p_C - \bar{r}^p_D\rho}{\nu(1-\rho^2)\sigma^2}, \tag{70}$$

$$\theta^p_{F,D} = \frac{\Big(1 - \gamma_D - \rho(1-\gamma_C)\Big)\sum_i \psi_i f'(G^p) + \bar{r}^p_D - \bar{r}^p_C\rho}{\nu(1-\rho^2)\sigma^2}. \tag{71}$$

□

A.9 Proof of Proposition 5 (Implications of the Planner's equilibrium)

The optimality conditions in Proposition 4 can be written in matrix form as below:

$$\bar{r}^p = \nu\Sigma\Theta - 2(1-\gamma)\psi f'\left(G^p\right). \tag{72}$$

Recall that $\psi \equiv \frac{1}{2}\sum_i \psi_i$. Premultiplying $\bar{r}$ by Θ' gives the market return yields:

$$\bar{r}^p_M = \nu\sigma^2_M - 2(1 - \sum_n \theta^p_n\gamma_n)\psi f'\left(G^p\right). \tag{73}$$

Combining the above two equations, we obtain:

$$\bar{r}^p = \frac{\Sigma\Theta - 2\nu^{-1}(1-\gamma)\psi f'\left(G^p\right)}{\sigma^2_M - 2\nu^{-1}(1 - \sum_n \theta^p_n\gamma_n)\psi f'\left(G^p\right)}\bar{r}_M, \tag{74}$$

which is equivalent to:

$$\bar{r}_n^p = \frac{Cov(\tilde{r}_M, \tilde{r}_n) - 2\nu^{-1}\psi(1-\gamma_n)f'(G^p)}{Var(\tilde{r}_M) - 2\nu^{-1}\psi(1-\sum_n \theta_n^p \gamma_n)f'(G^p)}\bar{r}_M. \tag{75}$$

The total wealth going to clean firms amounts to (based on Proposition 4):

$$\sum_i \theta_{i,C}^p \omega_0 = \Big(2(1-\gamma_C-\rho(1-\gamma_D))\sum_i \psi_i f'(G^p)+2\bar{r}_C^p-2\bar{r}_D^p\rho\Big)\Big(\nu(1-\rho^2)\sigma^2\Big)^{-1}\omega_0, \tag{76}$$

whereas in the private equilibrium, the total wealth going to clean firms amounts to:

$$\sum_i \theta_{i,C}\omega_0 = \Big((1-\gamma_C-\rho(1-\gamma_D))\sum_i \psi_i f'(G)+2\bar{r}_C-2\bar{r}_D\rho\Big)(\nu(1-\rho^2)\sigma^2)^{-1}\omega_0. \tag{77}$$

Note that:

$$\begin{aligned} G^p &= \omega_0(\sum_i \theta_{i,C}^p(1-\gamma_C) + (2-\sum_i \theta_{i,C}^p)(1-\gamma_D)), \text{ and} \\ G &= \omega_0(\sum_i \theta_{i,C}(1-\gamma_C) + (2-\sum_i \theta_{i,C})(1-\gamma_D)). \end{aligned}$$

Given $\gamma_D > \gamma_C$, as long as $\sum_i \theta_{i,C}^p > \sum_i \theta_{i,C}$, we can establish that $G^p > G$. Controlling for price effects of expected returns, from (76) and (77), $\sum_i \theta_{i,C}^p > \sum_i \theta_{i,C}$ holds. □

A.10 Proof of Lemma 2 (Taxes don't alter investor allocations)

Similar to Proposition 1, the optimality conditions for $\theta_{i,n}$ look exactly identical as before (see Equations (78) and (79)) except here G is a function of taxes.

$$\theta_{i,C} = \frac{\Big(1-\gamma_C-\rho(1-\gamma_D)\Big)\psi_i f'(G(\tau)) + \bar{r}_C - \bar{r}_D\rho}{\nu(1-\rho^2)\sigma^2}, \tag{78}$$

$$\theta_{i,D} = \frac{\Big(1-\gamma_D-\rho(1-\gamma_C)\Big)\psi_i f'(G(\tau)) + \bar{r}_D - \bar{r}_C\rho}{\nu(1-\rho^2)\sigma^2}. \tag{79}$$

Let us suppose $f''(G) = 0$, then $f'(G)$ is a constant, so the expressions for asset allocations are exactly the same as the case without taxes, and thus, asset allocations are unaffected by taxes.

However, if $f''(G) < 0$, $f'(G)$ depends on taxes, so in this case, we denote allocations as $\theta_{i,n}(\tau)$. $\square$

A.11 Proof of Proposition 6 (Crowding out)

We have already shown in the text that $G(\tau) - G > 0$ *iff* $\lambda < \gamma_D - (\gamma_D - \gamma_C)\theta_C$. Since $\gamma_D - \gamma_C > 0$ and $\theta_C < 1$, it follows that $\gamma_D - (\gamma_D - \gamma_C)\theta_C > \gamma_C$. Thus, if $\lambda \le \gamma_C$, $G(\tau) - G > 0$ always holds; if $\lambda > \gamma_C$, then when $\theta_C < \frac{\gamma_D - \lambda}{\gamma_D - \gamma_C}$ (equivalent to $\lambda < \gamma_D - (\gamma_D - \gamma_C)\theta_C$), it must be that $G(\tau) - G > 0$; however, when $\theta_C > \frac{\gamma_D - \lambda}{\gamma_D - \gamma_C}$, it follows that $G(\tau) - G < 0$. $\square$

A.12 Green policy: subsidy

The government levies taxes τ from each investor to subsidize green investment, and as before, a waste cost of λ is incurred. Firm n's green investment is z_n, so it receives δz_n subsidy from the government. This gives the relationship between τ and δ as:

$$2\tau(1-\lambda) = \delta \sum_n z_n. \tag{80}$$

The total amount of capital available for production is thus $k_{\tau,n} = k_n + \delta z_n$. Firm n solves the following problem:

$$\underbrace{\max}_{z_n, k_n} \; E_0 \tilde{M} \tilde{\epsilon}_n (k_{\tau,n} - z_n)^{\gamma_n} z_n^{1-\gamma_n} - k_n,$$

subject to:

$$k_{\tau,n} = k_n + \delta z_n.$$

The FOC for k_n is:

$$\mathbb{E}_0(\tilde{M}\tilde{\epsilon}_n)\gamma_n(k_{\tau,n} - z_n)^{\gamma_n - 1} z_n^{1-\gamma_n} = 1.$$

The FOC for z_n is:

$$\gamma_n(\delta-1)\mathbb{E}_0(\tilde{M}\tilde{\epsilon}_n)(k_{\tau,n}-z_n)^{\gamma_n-1} z_n^{1-\gamma_n} + (1-\gamma_n)\mathbb{E}_0(\tilde{M}\tilde{\epsilon}_n)(k_{\tau,n}-z_n)_n^{\gamma} z_n^{-\gamma_n} = 0.$$

Combining the above FOCs:

$$\frac{k_n - (1-\delta)z_n}{z_n} = \frac{(1-\delta)\gamma_n}{1-\gamma_n},$$

This is equivalent to:

$$z_n = \frac{1-\gamma_n}{1-\delta} k_n. \tag{81}$$

From the above equation, we can derive the expected excess return with subsidy $\bar{r}_n(\delta)$ as $\bar{r}_n(\delta) = \mathbb{E}_0 \tilde{A}_n(\frac{1}{1-\delta})^{1-\gamma_n} - 1$. It follows immediately that for $\delta \in (0,1)$, $\bar{r}_n(\delta) > \bar{r}_n$, and $d(\frac{1+\bar{r}_n(\delta)}{1+\bar{r}_n})/d\gamma_n < 0$.

Moreover, the following market clearing conditions hold:

$$k_n = \sum_i \theta_{i,n}(\omega_0 - \tau),$$

This gives us:

$$\eta_{i,n} = \frac{1-\gamma_n}{1-\delta}. \tag{82}$$

Thus, the subsidy δ also changes the public good loading terms in asset prices and allocations. First, we solve for investor i ($i \in (E,F)$)'s asset allocation $\theta_{i,C}$ and $\theta_{i,D}$:

$$\bar{r}_C + (1-\gamma_C)(1-\delta)^{-1}\psi_i f'(G) = \nu\sigma^2\theta_{i,C} + \nu\sigma^2\rho\theta_{i,D}, \tag{83}$$

$$\bar{r}_D + (1-\gamma_D)(1-\delta)^{-1}\psi_i f'(G) = \nu\sigma^2\theta_{i,D} + \nu\sigma^2\rho\theta_{i,C}. \tag{84}$$

Rearranging the above two equations in terms of asset allocations:

$$\theta_{i,C} = \frac{\Big(1-\gamma_C-\rho(1-\gamma_D)\Big)(1-\delta)^{-1}\psi_i f'(G) + \bar{r}_C - \bar{r}_D\rho}{\nu(1-\rho^2)\sigma^2}, \tag{85}$$

$$\theta_{i,D} = \frac{\Big(1-\gamma_D-\rho(1-\gamma_C)\Big)(1-\delta)^{-1}\psi_i f'(G) + \bar{r}_D - \bar{r}_C\rho}{\nu(1-\rho^2)\sigma^2}. \tag{86}$$

Let $\psi = \frac{\psi_C+\psi_D}{2}$, we combine investors' FOCs for asset allocations to produce:

$$\bar{r}_C + (1-\gamma_C)(1-\delta)^{-1}\psi f'(G) = \nu\sigma^2(\sum_{i\in(E,F)}\theta_{i,C}/2 + \rho\sum_{i\in(E,F)}\theta_{i,D}/2), \tag{87}$$

$$\bar{r}_D + (1-\gamma_D)(1-\delta)^{-1}\psi f'(G) = \nu\sigma^2(\sum_{i,\in(E,F)}\theta_{i,D}/2 + \rho\sum_{i,\in(E,F)}\theta_{i,C}/2). \tag{88}$$

The above two equations in matrix form are equivalent to:

$$\bar{r} = \nu\Sigma\Theta - \frac{1-\gamma}{1-\delta}\psi f'(G). \tag{89}$$

As before, we can derive the CAPM-like formula as follows, which is modified by the subsidy δ. The expected excess returns of firm n in equilibrium are expressed as:

$$\bar{r}_n = \beta_n^{G(\delta)}\bar{r}_M, \tag{90}$$

where:

$$\beta_n^{G(\delta)} = \frac{Cov(\tilde{r}_M, \tilde{r}_n) - \nu^{-1}(1-\delta)^{-1}(1-\gamma_n)\psi f'(G)}{\sigma_M^2 - \nu^{-1}(1-\sum_n \theta_n\gamma_n)(1-\delta)^{-1}\psi f'(G)}. \tag{91}$$

Although the CAPM-like formula changes because of δ, with some algebra the asset pricing implication of condition (14) remains the same.

The public good G in this case can be expressed as:

$$G = \frac{1-\gamma_C}{1-\delta}(\theta_{EC}+\theta_{FC})(\omega_0 - \frac{\delta G}{2(1-\lambda)}) + \frac{1-\gamma_D}{1-\delta}(\theta_{ED}+\theta_{FD})(\omega_0 - \frac{\delta G}{2(1-\lambda)}). \tag{92}$$

Note that $\theta_C = \frac{\theta_{EC}+\theta_{FC}}{2}$, and $\theta_{ED}+\theta_{FD} = 2-(\theta_{EC}+\theta_{FC})$, we can rewrite the above equation as:

$$G = \frac{\omega_0}{\frac{1-\delta}{2(1-\gamma_D+(\gamma_D-\gamma_C)\theta_C)} + \frac{\delta}{2(1-\lambda)}}. \tag{93}$$

Let $\Xi \equiv \frac{1-\delta}{2(1-\gamma_D+(\gamma_D-\gamma_C)\theta_C)}$ and $\Phi \equiv \frac{\delta}{2(1-\lambda)}$, we have $\frac{d\Phi}{d\delta} = \frac{1}{2(1-\gamma)} > 0$ and $\frac{d\Xi}{d\delta} = \frac{-(1-\gamma_D+(\gamma_D-\gamma_C)\theta_C)-(1-\delta)(\gamma_D-\gamma_C)\frac{d\theta_C}{d\delta}}{2(1-\gamma_D+(\gamma_D-\gamma_C)\theta_C)^2}$. Now we identify a sufficient condition such that $d\Xi/d\delta < 0$. Suppose the semi-elasticity $\epsilon_{\theta_C,\delta} \equiv \frac{d\theta_C}{\theta_C d\delta}$ satisfies $\epsilon_{\theta_C,\delta} > \frac{1}{\sigma-1}$, then it follows that:

$$-\frac{d\theta_C}{d\delta}(1-\delta)(\gamma_D-\gamma_C) < \theta_C(\gamma_D-\gamma_C),$$

which is sufficient for $d\Xi/d\delta < 0$. □

A.13 Negative Externalities

Here we assume the dirty firm's investment z_D contributes negatively to the public good:

$$G = z_C - z_D. \tag{94}$$

Firm D's maximization problem remains the same as before, and in equilibrium $z_D = (1-\gamma_D)k_D$. The only difference is that $\partial G/\partial z_D = -1$, instead of $\partial G/\partial z_D = 1$. We have $\eta_D = \gamma_D - 1$, which is a negative number.

Because the dirty firm does not contribute to G positively, we no longer need to assume $\gamma_C < \gamma_D$, only that $\gamma_D, \gamma_C \leq 1$.

Investors' asset holdings become:

$$\theta_{E,C} = \frac{\Big(1 - \gamma_C - \rho(\gamma_D - 1)\Big)\psi_E f'(G) + \bar{r}_C - \bar{r}_D \rho}{\nu(1-\rho^2)\sigma^2}, \tag{95}$$

$$\theta_{E,D} = \frac{\Big(\gamma_D - 1 - \rho(1 - \gamma_C)\Big)\psi_E f'(G) + \bar{r}_D - \bar{r}_C \rho}{\nu(1-\rho^2)\sigma^2}, \tag{96}$$

$$\theta_{F,C} = \frac{\Big(1 - \gamma_C - \rho(\gamma_D - 1)\Big)\psi_F f'(G) + \bar{r}_C - \bar{r}_D \rho}{\nu(1-\rho^2)\sigma^2}, \tag{97}$$

$$\theta_{F,D} = \frac{\Big(\gamma_D - 1 - \rho(1 - \gamma_C)\Big)\psi_F f'(G) + \bar{r}_D - \bar{r}_C \rho}{\nu(1-\rho^2)\sigma^2}. \tag{98}$$

We can see that investor E invests more in the clean firm and less in the dirty firm than investor F. This result is unambiguous, in contrast to our main model result.

Moreover, our main model result suggests that investor E is always invests more than investor F, but in the case with the negative externality, this does not always hold. Note that the difference in the amount of investment between investor E and investor F is:

$$\begin{aligned}
(\theta_{E,C} + \theta_{E,D} - \theta_{F,C} - \theta_{F,D})\omega_0 &= \omega_0(\nu(1-\rho^2)\sigma^2)^{-1}((1 - \gamma_C - \rho(\gamma_D - 1))f'(G)(\phi_E - \phi_F) \\
&\quad + (\gamma_D - 1 - \rho(1 - \gamma_C))f'(G)(\psi_E - \psi_F)) \\
&= \omega_0(\nu(1-\rho^2)\sigma^2)^{-1}(\gamma_D - \gamma_C)(1 - \rho)f'(G)(\psi_E - \psi_F).
\end{aligned}$$

Because $f'(G) > 0$ and $\psi_E - \psi_F > 0$, if $\gamma_D > \gamma_C$, then investor E still invests more in the stock market, as is the case in the main model. If $\gamma_D < \gamma_C$, then investor E invests less in the stock market than investor F. This is different from our main model result that investor E always invests more in the stock market.

In terms of asset prices:

$$\bar{r}_C + (1-\gamma_C)\psi_E f'(G) = \nu\sigma^2\theta_{E,C} + \nu\sigma^2\rho\theta_{E,D}, \tag{99}$$

$$\bar{r}_D + (\gamma_D-1)\psi_E f'(G) = \nu\sigma^2\theta_{E,D} + \nu\sigma^2\rho\theta_{E,C}, \tag{100}$$

$$\bar{r}_C + (1-\gamma_C)\psi_F f'(G) = \nu\sigma^2\theta_{F,C} + \nu\sigma^2\rho\theta_{F,D}, \tag{101}$$

$$\bar{r}_D + (\gamma_D-1)\psi_F f'(G) = \nu\sigma^2\theta_{F,D} + \nu\sigma^2\rho\theta_{F,C}. \tag{102}$$

$$\bar{r}_C + (1-\gamma_C)\psi f'(G) = \nu\sigma^2\frac{\theta_{E,C}+\theta_{F,C}}{2} + \nu\sigma^2\rho\frac{\theta_{E,D}+\theta_{F,D}}{2}, \tag{103}$$

$$\bar{r}_D + (\gamma_D-1)\psi f'(G) = \nu\sigma^2\frac{\theta_{E,D}+\theta_{F,D}}{2} + \nu\sigma^2\rho\frac{\theta_{E,C}+\theta_{F,C}}{2}. \tag{104}$$

Note that $\psi \equiv 1/2\sum_i \psi_i$, $\Theta \equiv [\theta_C, \theta_D]'$, $\theta_C = \frac{\theta_{E,C}+\theta_{F,C}}{2}$, $\theta_D = \frac{\theta_{E,D}+\theta_{F,D}}{2}$, and $\bar{r} \equiv [\bar{r}_C, \bar{r}_D]'$. Equations (103) (104) in matrix form are equivalent to:

$$\bar{r} = \nu\Sigma\Theta - \hat{\gamma}\psi f'(G), \tag{105}$$

where $\hat{\gamma} \equiv [1-\gamma_C, \gamma_D - 1]'$.

Multiplying the above equation by Θ' gives the market equilibrium, $\bar{r}_M = \Theta'\bar{r}$:

$$\bar{r}_M = \nu\sigma_M^2 - \Big(\theta_C(1-\gamma_C) + \theta_D(\gamma_D-1)\Big)\psi f'(G). \tag{106}$$

The expected excess return of firm C in equilibrium is:

$$\bar{r}_C = \beta_C^G \bar{r}_M, \tag{107}$$

where:

$$\beta_C^G = \frac{Cov(\tilde{r}_M, \tilde{r}_C) - \nu^{-1}(1-\gamma_C)\psi f'(G)}{\sigma_M^2 - \nu^{-1}\Big(\theta_C(1-\gamma_C) + \theta_D(\gamma_D-1)\Big)\psi f'(G)}. \tag{108}$$

The expected excess return of firm D in equilibrium is:

$$\bar{r}_D = \beta_D^G \bar{r}_M, \tag{109}$$

where:

$$\beta_D^G = \frac{Cov(\tilde{r}_M, \tilde{r}_D) - \nu^{-1}(\gamma_D - 1)\psi f'(G)}{\sigma_M^2 - \nu^{-1}\Big(\theta_C(1-\gamma_C) + \theta_D(\gamma_D - 1)\Big)\psi f'(G)}. \tag{110}$$

Suppose $\bar{r}_M > 0$, which means the denominator of β_n^G is positive. We can express alpha as:

$$\alpha_n = (\beta_n^G - \beta_n)\bar{r}_M. \tag{111}$$

Whether alpha is positive or negative depends on $\beta_n^G - \beta_n$. Let $X = \sigma_M^2 - \nu^{-1}\Big(\theta_C(1-\gamma_C) + \theta_D(\gamma_D - 1)$, and $X > 0$. With some algebra, we show:

$$(\beta_C^G - \beta_C)X\sigma_M^2 = -\nu^{-1}\psi f'(G)\Big((1-\gamma_C)\sigma_M^2 - Cov(\tilde{r}_M, \tilde{r}_C)(\theta_C(1-\gamma_C) + \theta_D(\gamma_D - 1))\Big). \tag{112}$$

Suppose $\alpha_C < 0$, then from (112):

$$1 - \gamma_C > \beta_C(\theta_C(1-\gamma_C) + \theta_D(\gamma_D - 1)). \tag{113}$$

It follows that, focusing on positive $\bar{r}_M$, the clean firm exhibits a negative alpha *iff* $\frac{1-\gamma_C}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} > \beta_C$, and a positive alpha *iff* $\frac{1-\gamma_C}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} < \beta_C$.

Turning to the dirty firm, we can derive a similar equation:

$$(\beta_D^G - \beta_D)X\sigma_M^2 = -\nu^{-1}\psi f'(G)\Big((\gamma_D - 1)\sigma_M^2 - Cov(\tilde{r}_M, \tilde{r}_D)(\theta_C(1-\gamma_D) + \theta_D(\gamma_D - 1))\Big). \tag{114}$$

Similarly, we can show that the dirty firm exhibits a positive alpha *iff* $\frac{\gamma_D-1}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} < \beta_D$, and a negative alpha *iff* $\frac{\gamma_D-1}{\theta_C(1-\gamma_C)+\theta_D(\gamma_D-1)} > \beta_D$.
$\square$

A.14 Proof of Lemma 3 (Donation choice)

Let d_i be investor i ($i \in (E, F)$)'s donation to public goods. Investor i's utility function can be rewritten as $\mathbb{E}_0(\tilde{\omega}_i) - \nu Var(\tilde{\omega}_i)/(2(\omega_0 - d_i)) + \psi_i f(G)$, where:

$$\mathbb{E}_0\tilde{\omega}_i = (\omega_0 - d_i)(1 + r_f + \bar{r}_C\theta_{i,C} + \bar{r}_D\theta_{i,D}), \quad (115)$$

$$Var\tilde{\omega}_i = (\omega_0 - d_i)^2(\sigma^2\theta_{i,C}^2 + \sigma^2\theta_{i,D}^2 + 2\theta_{i,C}\theta_{i,D}\sigma^2\rho), \quad (116)$$

$$G = \sum_n z_n + \sum_{i\in(E,F)} d_i \quad (117)$$

and

$$d_i \geq 0. \quad (118)$$

Let ϕ_i be the Lagrangian multiplier of $-d_i \leq 0$, the K-T complementary slackness condition is $d_i \geq 0$, $\phi_i \geq 0$, and $d_i\phi_i = 0$, and note that $\sum_n z_n = \sum_{i\in(E,F)}(\omega_0 - d_i)\sum_n((1-\gamma_n)\theta_{i,n})$.

Writing the FOC with respect to d_i gives us:

$$\frac{\nu}{2}\sigma_i^2 + \psi_i f'(G)(1 - \sum_n(1-\gamma_n)\theta_{i,n}) + \phi_i = 1 + r_f + \sum_n \bar{r}_n\theta_{i,n}, \quad (119)$$

where $\sigma_i^2 \equiv \sigma^2\theta_{i,C}^2 + \sigma^2\theta_{i,D}^2 + 2\theta_{i,C}\theta_{i,D}\sigma^2\rho$. The left-hand side of the above equation expresses the marginal benefits of contributing wealth directly to G, and the right-hand side expresses the marginal cost. If $\phi_i > 0$, then $d_i = 0$. $\square$

A.15 Proof of Proposition 9(Amount of public good with donations)

Let d_i be the amount of donation chosen by investor i, and $d_i \geq 0$. Note that donations do not affect asset prices and allocations when $f''(G) = 0$. The total amount of public goods with donations is:

$$G(d) = \sum_i d_i + \sum_i(\omega_0 - d_i)\theta_{i,C}(1-\gamma_C) + \sum_i(\omega_0 - d_i)\theta_{i,D}(1-\gamma_D), \quad (120)$$

and without donations, it is:

$$G = \sum_i \omega_0\theta_{i,C}(1-\gamma_C) + \sum_i \omega_0\theta_{i,D}(1-\gamma_D). \quad (121)$$

It follows that:

$$G(d) - G = \sum_i \Big(d_i - d_i\theta_{i,C}(1-\gamma_C) - d_i\theta_{i,D}(1-\gamma_D) \Big). \tag{122}$$

Since $\gamma_D > \gamma_C > \frac{1}{2}$ is assumed in the main model's numerical analysis, $1-\gamma_D < 1-\gamma_C < \frac{1}{2}$, and $\theta_{i,n} \leq 1$, it follows that $G(d) - G \geq 0$. This expression is strictly larger than zero when at least one $d_i > 0$ and equal to zero when both d_i are equal to zero.

□

A.16 Risky Public Good

Investor i ($\in (E,F)$)'s utility function can be rewritten as:

$$\mathbb{E}_0(\tilde{\omega}_i) + \psi_i \mathbb{E}_0(\tilde{G}) - \frac{\nu}{2\omega_0}\Big(Var(\tilde{\omega}_i) + \psi_i^2 Var(\tilde{G}) + 2\psi_i Cov(\tilde{\omega}_i, \tilde{G}) \Big) \tag{123}$$

where:

$$\tilde{G} = \tilde{\epsilon}_G \omega_0((1-\gamma_C)\sum_{i\in(E,F)} \theta_{i,C} + (1-\gamma_D)\sum_{i\in(E,F)} \theta_{i,D}, \tag{124}$$

and $\tilde{\epsilon}_G$ is the risk to public good production, with mean $\mathbb{E}_0(\tilde{\epsilon}_G) = 1$, variance σ_G^2, and correlation with firm C denoted by ρ_{CG}, and with firm D denoted by ρ_{DG}. Moreover,

$$\mathbb{E}_0\tilde{\omega}_i = \omega_0(1 + r_f + \bar{r}_C\theta_{i,C} + \bar{r}_D\theta_{i,D}), \tag{125}$$

$$Var\tilde{\omega}_i = \omega_0^2\sigma^2\theta_{i,C}^2 + \omega_0^2\sigma^2\theta_{i,D}^2 + 2\theta_{i,C}\theta_{i,D}\omega_0^2\sigma^2\rho. \tag{126}$$

$$\mathbb{E}_0(\tilde{G}) = \omega_0\Big((1-\gamma_C)\sum_{i\in(E,F)} \theta_{i,C} + (1-\gamma_D)\sum_{i\in(E,F)} \theta_{i,D}\Big) \tag{127}$$

$$Var(\tilde{G}) = \sigma_G^2 \omega_0^2 \Big((1-\gamma_C) \sum_{i \in (E,F)} \theta_{i,C} + (1-\gamma_D) \sum_{i \in (E,F)} \theta_{i,D} \Big)^2 \tag{128}$$

$$\begin{aligned} Cov(\tilde{\omega}_E, \tilde{G}) = \rho_{CG}\sigma_G\sigma\omega_0^2 (\Big(1-\gamma_C)\theta_{i,C} \sum_{i \in (E,F)} \theta_{i,D} + (1-\gamma_D)\theta_{i,C} \sum_{i \in (E,F)} \theta_{i,D} \Big) \\ + \rho_{DG}\sigma_G\sigma\omega_0^2 (\Big(1-\gamma_C)\theta_{i,D} \sum_{i \in (E,F)} \theta_{i,C} + (1-\gamma_D)\theta_{i,D} \sum_{i \in (E,F)} \theta_{i,D} \Big) \end{aligned}$$

The FOC for $\theta_{i,C}, \theta_{i,D}$ are:

$$\bar{r}_C + (1-\gamma_C)\psi_i \Phi(G; iC) = \nu\sigma^2\theta_{i,C} + \nu\sigma^2\rho\theta_{i,D}, \tag{129}$$

$$\bar{r}_D + (1-\gamma_D)\psi_i \Phi(G; iD) = \nu\sigma^2\theta_{i,D} + \nu\sigma^2\rho\theta_{i,C}, \tag{130}$$

where for $i \in (E, F)$:

$$\Phi(G;EC) = 1 - \frac{\nu}{\omega_0}\sigma_G^2\psi_E\mathbb{E}_0(\tilde{G}) - \nu\rho_{CG}\sigma\sigma_G(2\theta_{EC} + \theta_{FC} + \frac{1-\gamma_D}{1-\gamma_C}(\theta_{ED} + \theta_{FD})) - \nu\rho_{DG}\sigma\sigma_G\theta_{ED}, \tag{131}$$

$$\Phi(G;ED) = 1 - \frac{\nu}{\omega_0}\sigma_G^2\psi_E\mathbb{E}_0(\tilde{G}) - \nu\rho_{DG}\sigma\sigma_G(2\theta_{ED} + \theta_{FD} + \frac{1-\gamma_C}{1-\gamma_D}(\theta_{EC} + \theta_{FC})) - \nu\rho_{CG}\sigma\sigma_G\theta_{EC}. \tag{132}$$

$$\Phi(G;FC) = 1 - \frac{\nu}{\omega_0}\sigma_G^2\psi_F\mathbb{E}_0(\tilde{G}) - \nu\rho_{CG}\sigma\sigma_G(2\theta_{FC} + \theta_{EC} + \frac{1-\gamma_D}{1-\gamma_C}(\theta_{ED} + \theta_{FD})) - \nu\rho_{DG}\sigma\sigma_G\theta_{FD}, \tag{133}$$

$$\Phi(G;FD) = 1 - \frac{\nu}{\omega_0}\sigma_G^2\psi_F\mathbb{E}_0(\tilde{G}) - \nu\rho_{DG}\sigma\sigma_G(2\theta_{FD} + \theta_{ED} + \frac{1-\gamma_C}{1-\gamma_D}(\theta_{EC} + \theta_{FC})) - \nu\rho_{CG}\sigma\sigma_G\theta_{FC}. \tag{134}$$

We now combine investors' date $t = 0$ budget constraints with the market clearing condition for the risk-free asset. It follows that:

$$\theta_{E,C} + \theta_{E,D} + \theta_{F,C} + \theta_{F,D} = 2. \tag{135}$$

We use Equations (127), (129), (130), (131), (132), (133), (134), (135), together with $\bar{r}_C = A_C - r_f - 1$ and $\bar{r}_D = A_D - r_f - 1$ to solve for $\theta_{i,C}, \theta_{i,D}(i \in (E, F))$,$r_f$,$\bar{r}_C$,$\bar{r}_D$ and $\Phi(G; EC)$,$\Phi(G, ED)$, $\Phi(G, FC)$,$\Phi(G, FD)$,$\mathbb{E}_0(\tilde{G})$ numerically, and the solutions are used in the simulations for Figure 13.

A.17 Tables and Figures

Parameter	Description	Base Case	Range
ψ_E	Investor E's weight on the public good	0.8	$[0.4, 1.6]$
ψ_F	Investor F's weight on the public good	0.3	$[0, 0.7]$
ν	Risk aversion coefficient	4	$[2, 6]$
ρ	Correlation between the returns of C and D	0.2	$[-0.4, 0.9]$
γ_C	Firm C output elasticity of capital	0.6	
γ_D	Firm D output elasticity of capital	0.9	
σ	Standard deviation of returns	0.3	
ω_0	Initial wealth for each investor	1	
$E[\tilde{A}_n]$	Expected productivity (for $n = \{C, D\}$)	1.14	
λ	Government waste	0.2	$[0, 0.8]$

Table 1: This table shows the base values of the exogenous model parameters for our numerical examples. The last column shows the range of values considered in the sensitivity analyses.

Comments on the simulation:

- We use the functional form $f(G) = \sqrt{G}$.
- The model assumes that $\psi_E > \psi_F \geq 0$. We limit ψ_E to be below 1.6 in the benchmark, but in a few instances we will increase it to 4 to explore the results further.
- The risk aversion coefficient ν is consistent with the literature (e.g., Bodie et al. (2020)).
- We use a positive correlation for our benchmark, consistent with estimates in Pollet and Wilson (2010). Our benchmark value is higher than theirs (.024), but correlation is one of the variables we examine over a large range.
- Also from the model, we assume that $0 \leq \gamma_C < \gamma_D \leq 1$. Furthermore, in the text we assume that $\gamma_C > \frac{1}{2}$ in order to ensure that both firms

focus on production of the private good, although sometimes we will look at the value $\gamma_C = 0.2$ to explore the results further.

- We select a volatility of 0.3. Berk and DeMarzo (2019) show that the S&P 500 historical volatility is 0.2 and the individual constituents of the S&P 500 have historical volatility above 0.25.

- The effective productivity shock is set to calibrate the risk-free rate to be approximately 2%.

- The parameter ranges for the risk aversion and correlation parameters are chosen such that the overall market excess return r_M is positive. Moreover, correlations below -40% would be highly unrealistic.

- The parameter λ captures a government waste cost, namely, for τ amount of taxes levied, only $(1-\lambda)\tau$ effectively contribute to public goods provision. Bandiera et al. (2009), considering both average passive and active government's waste, estimate a percentage waste between 17% and 29%. Our base case value of 0.2 is well within this range and closer to the lower bound to be conservative.

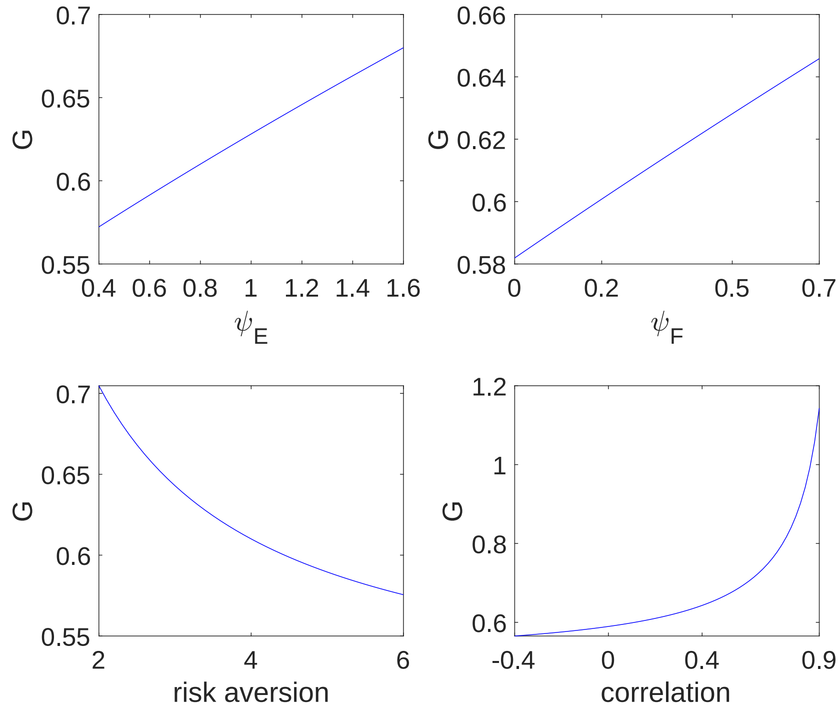

Figure 1: **Sensitivity of Public Good Provision to Key Parameters**. This Figure displays the sensitivity of public good provision to key parameters, namely ψ_E (top left panel), ψ_F (top right panel), ν (bottom left panel) and ρ (bottom right panel). The base values of the parameters are presented in Table 1, which also specifies the ranges considered for each parameter.

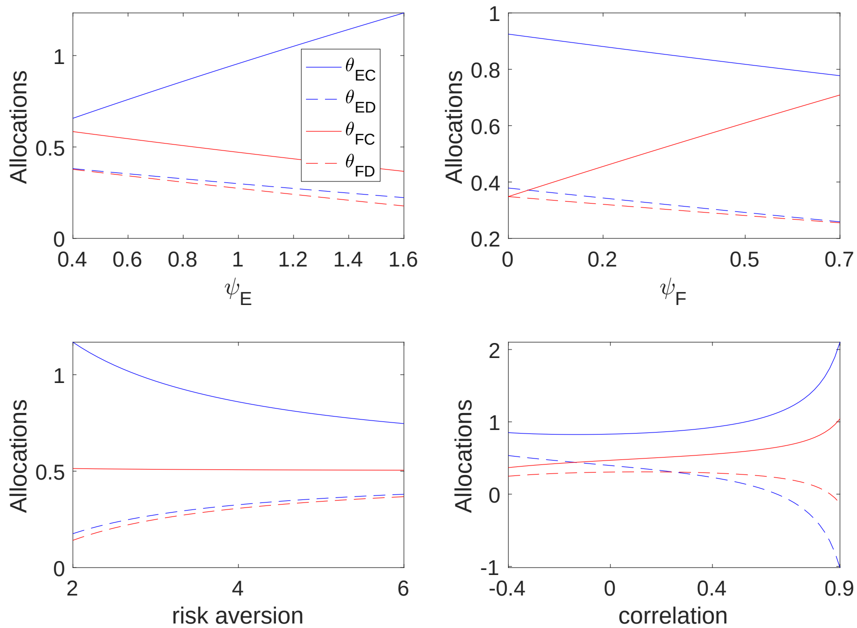

Figure 2: **Sensitivity of Investors' Allocations to Key Parameters**. This Figure displays the sensitivity of the two investors' allocations to the clean and dirty firms to key parameters, namely ψ_E (top left panel), ψ_F (top right panel), ν (bottom left panel) and ρ (bottom right panel). The base values of the parameters are presented in Table 1, which also specifies the ranges considered for each parameter.

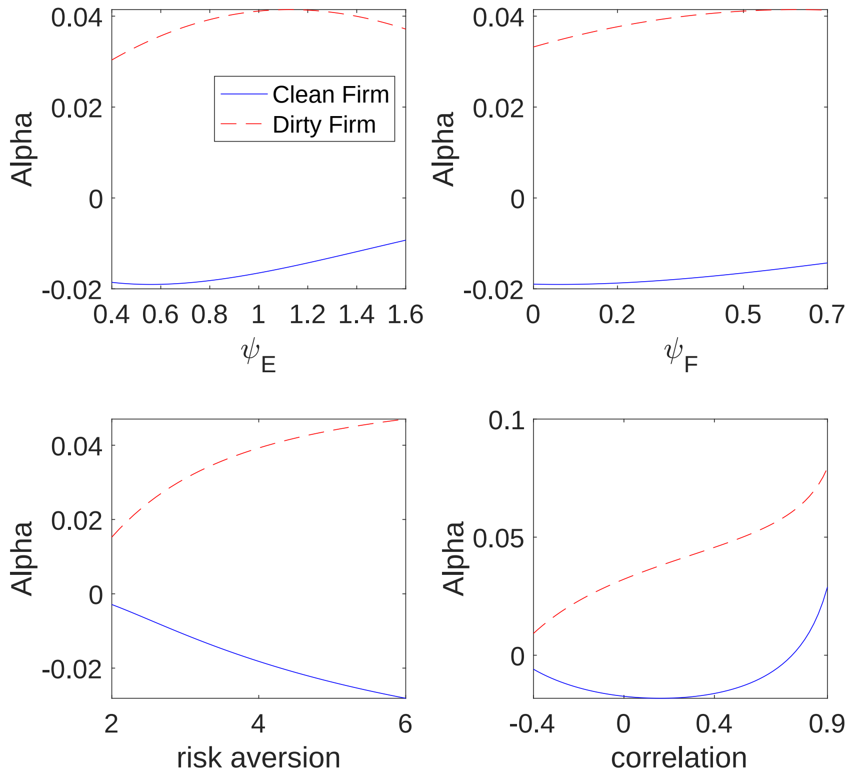

Figure 3: **Sensitivity of Stocks' Alphas to Key Parameters**.
This Figure displays the sensitivity of the alpha of the two stocks to key parameters, namely ψ_E (top left panel), ψ_F (top right panel), ν (bottom left panel) and ρ (bottom right panel). The base values of the parameters are presented in Table 1, which also specifies the ranges considered for each parameter.

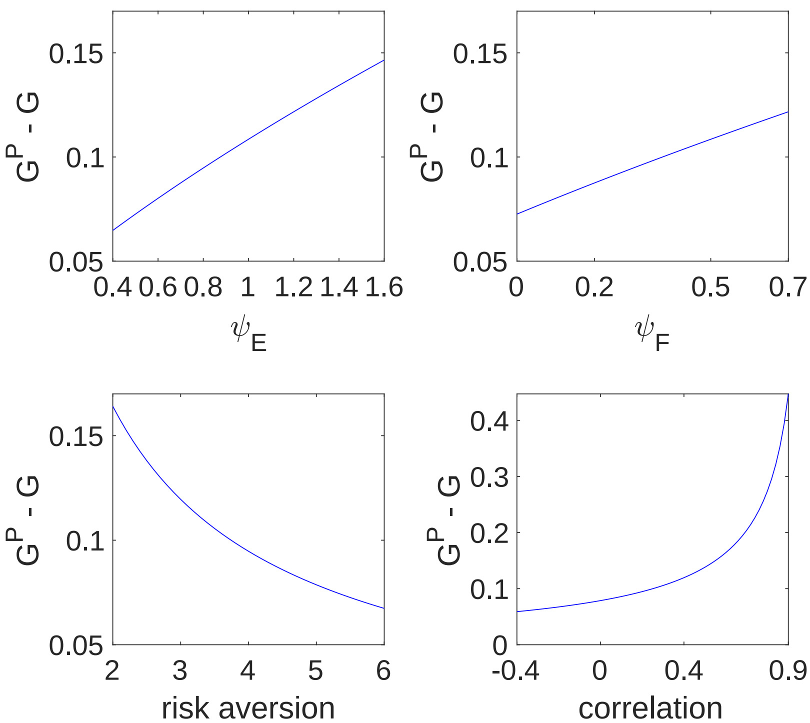

Figure 4: **Sensitivity of Free Riding Problem to Key Parameters**. This Figure displays the difference between public good provisions in the planner and market equilibria, as a function of key parameters, namely ψ_E (top left panel), ψ_F (top right panel), ν (bottom left panel) and ρ (bottom right panel). The base values of the parameters are presented in Table 1, which also specifies the ranges considered for each parameter.

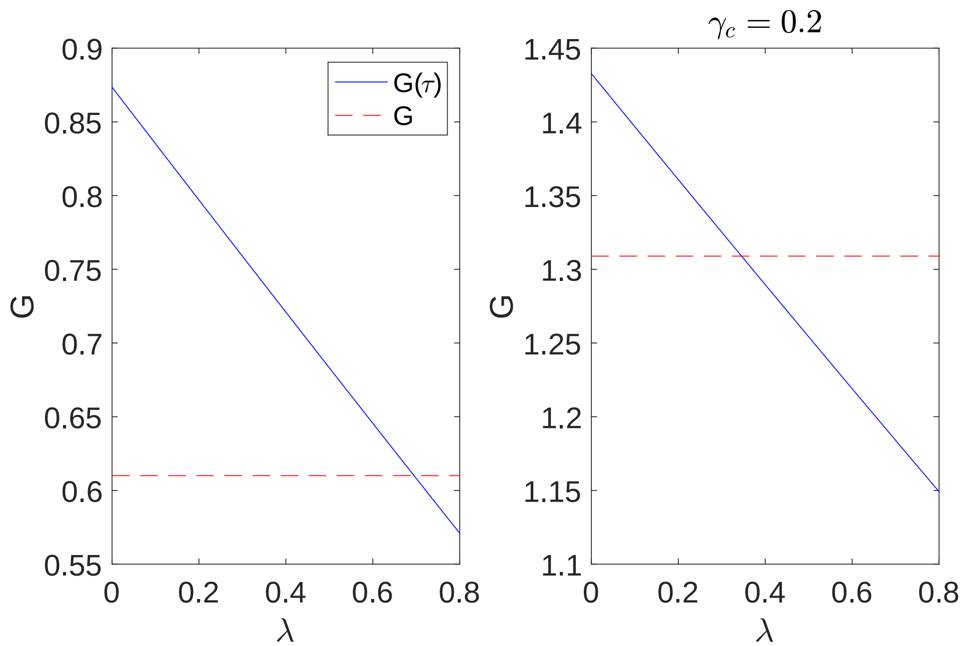

Figure 5: **Sensitivity of Crowding out to λ.**
This Figure displays how changes in λ (the waste in government spending) affect the provision of public good relative to the model without taxation when we use our benchmark value of γ_C (left panel) and when we use a low value of γ_C (right panel). We set the tax rate to $\tau = 0.2$ as our benchmark value. The other base values of the parameters are presented in Table 1.

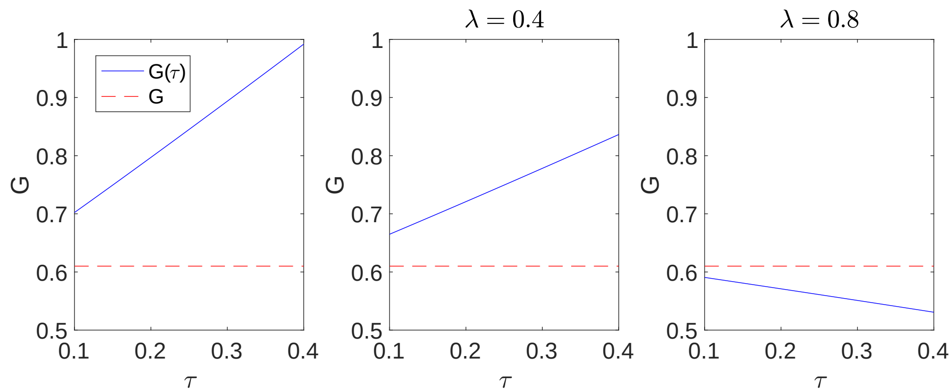

Figure 6: **Sensitivity of Crowding out to τ.**
This Figure displays how changes in the tax rate τ affect the provision of public good relative to the model without taxation when we use our benchmark value of λ (left panel) and when we use a high value of λ (middle panel); lastly we set λ to 0.8 (right panel). The base values of the parameters are presented in Table 1.

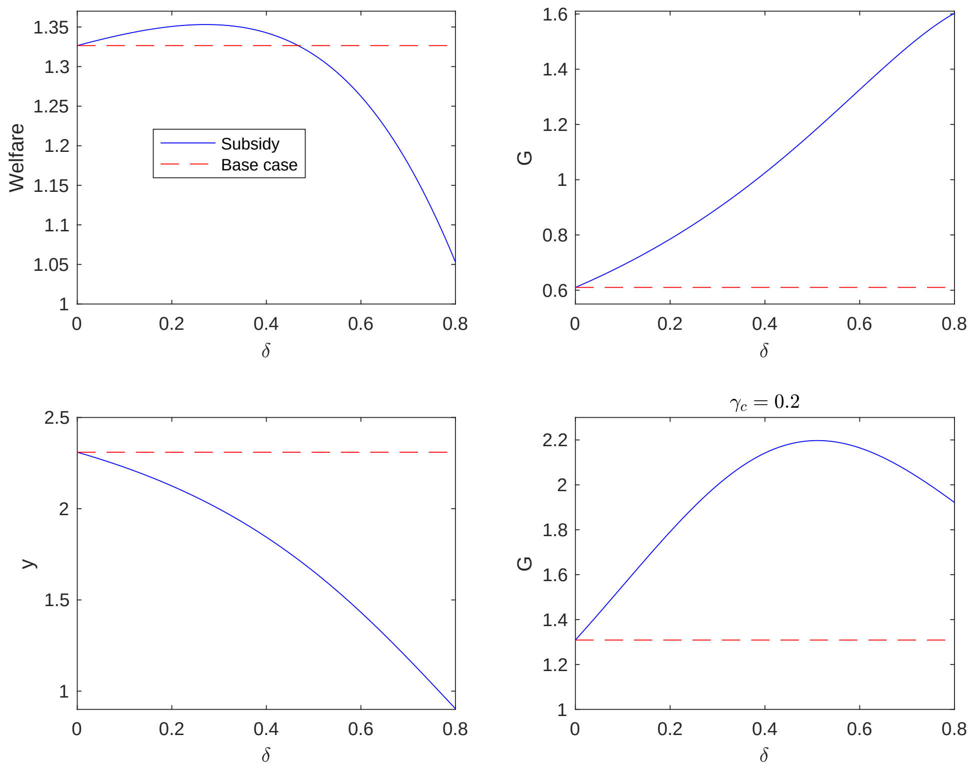

Figure 7: **Social Welfare and Allocations with or without Subsidy**. This Figure displays, for the benchmark value of γ_C, how the changes in green subsidy δ affect social welfare (top left panel), public goods provision (top right panel) and expected total private goods provision (denoted as $y = y_C + y_D$, bottom left panel) relative to the benchmark model without subsidy, and the public goods provision when we use a low value of γ_C (bottom right panel).

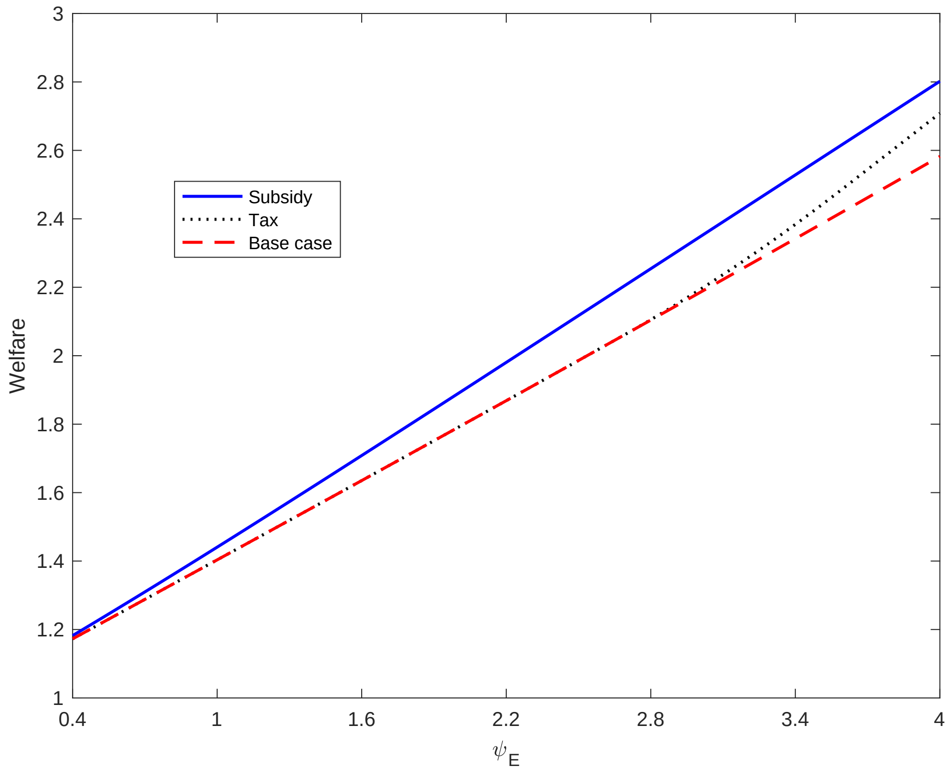

Figure 8: **Social Welfare with Optimal Green Subsidy and Optimal Tax**.

This Figure displays the social welfare corresponding to the optimal green subsidy (solid line) relative to the social welfare corresponding to optimal public provision of public goods using taxes (dotted line), while varying investor preference ψ_E. The dashed line corresponds to the welfare of the benchmark case. The base values of the parameters are presented in Table 1. However, the range of ψ_E has been extended to $[0.4, 4]$.

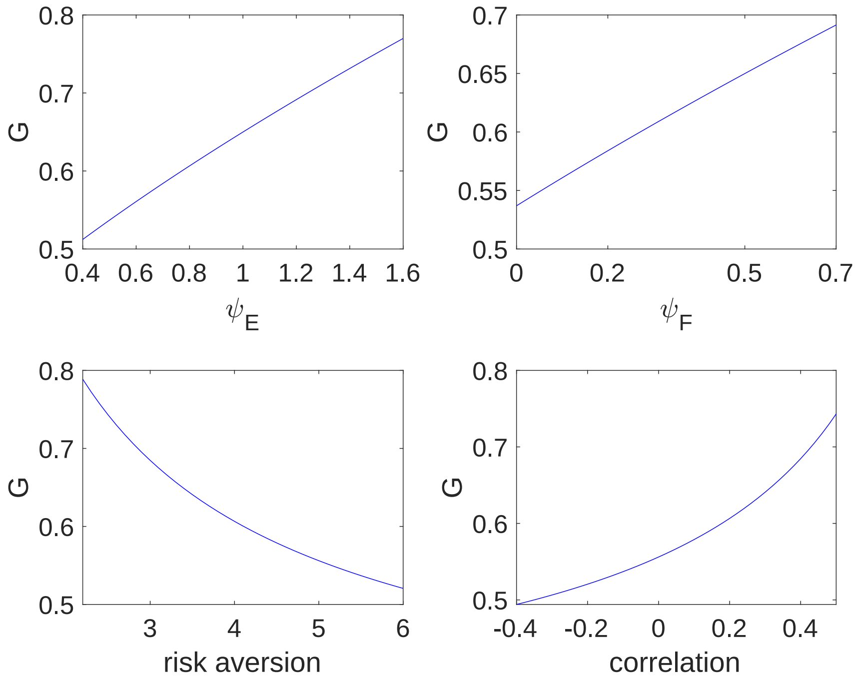

Figure 9: **Negative Externality Case: Sensitivity of Public Good Provision to Key Parameters**.
This Figure displays the sensitivity of public good provision in the negative externality case to key parameters, namely ψ_E (top left panel), ψ_F (top right panel), ν (bottom left panel) and ρ (bottom right panel). The base values of the parameters are presented in Table 1. However, the ranges of two of the panels are changed: the range of risk aversion parameter ν is set to $[2.2, 6]$ and the range of correlation ρ is set to $[-0.4, 0.5]$. Both of these ranges are smaller than the ranges in Table 1 to exclude cases of the market taking a net short-sale position of the dirty firm.

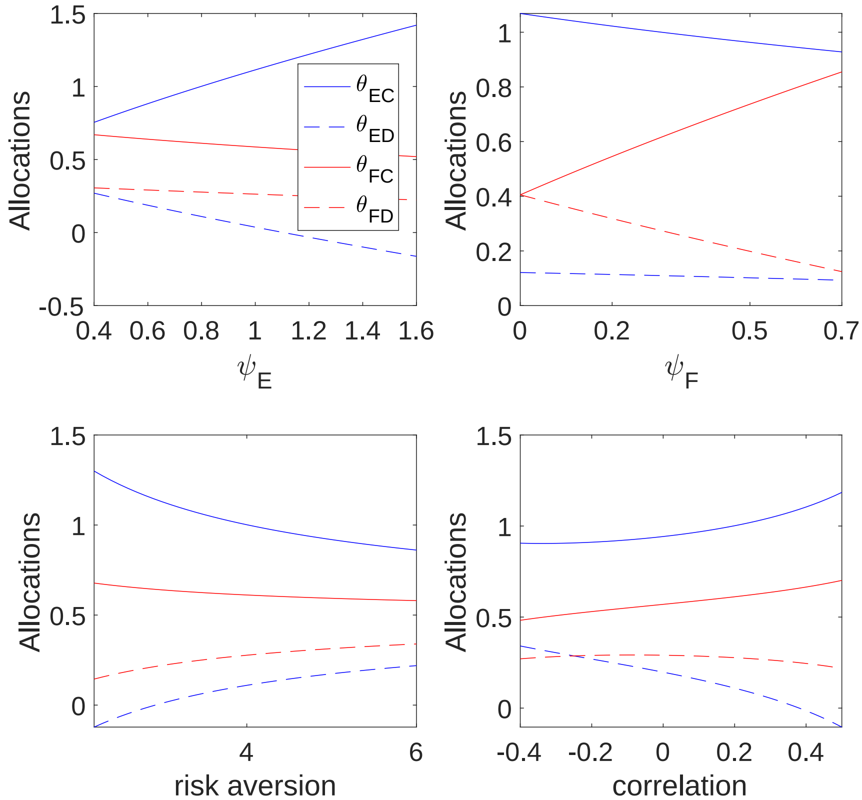

Figure 10: **Negative Externality Case: Sensitivity of Investors Allocations to Key Parameters**.
This Figure displays the sensitivity of the two investors' allocations to the clean and dirty firms in the negative externality case to key parameters, namely ψ_E (top left panel), ψ_F (top right panel), ν (bottom left panel) and ρ (bottom right panel). The base values of the parameters are presented in Table 1. However, the ranges of two of the panels are changed: the range of risk aversion parameter ν is set to $[2.2, 6]$ and the range of correlation ρ is set to $[-0.4, 0.5]$. Both of these ranges are smaller than the ranges in Table 1 to exclude cases of the market taking a net short-sale position of the dirty firm.

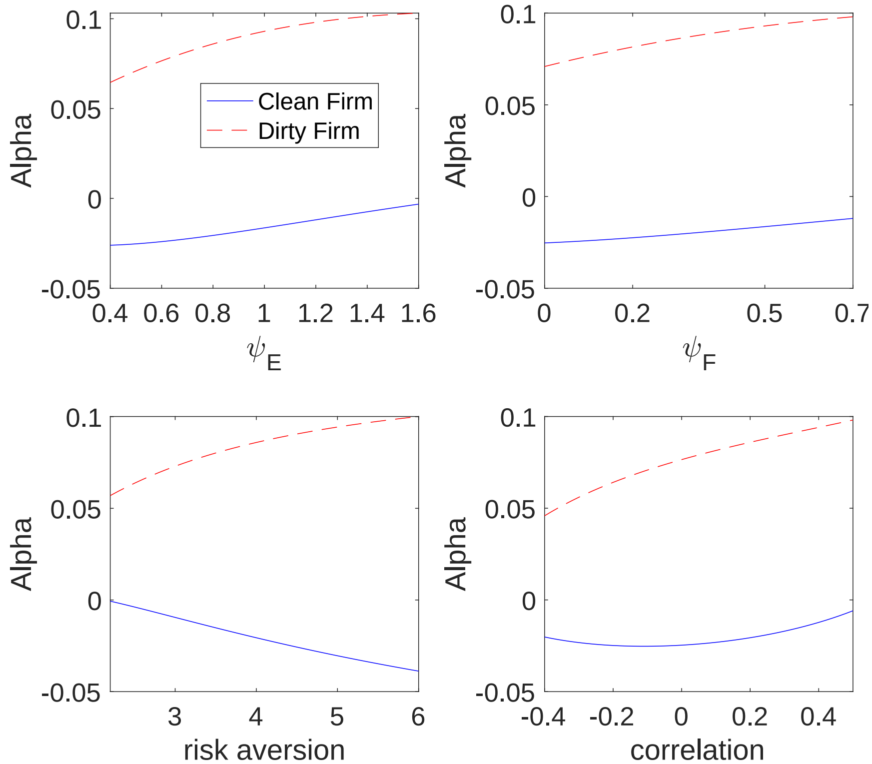

Figure 11: **Negative Externality Case: Sensitivity of Stocks' Alphas to Key Parameters**.
This Figure displays the sensitivity of the alpha of the two stocks in the negative externality case to key parameters, namely ψ_E (top left panel), ψ_F (top right panel), ν (bottom left panel) and ρ (bottom right panel). The base values of the parameters are presented in Table 1. However, the ranges of two of the panels are changed: the range of risk aversion parameter ν is set to $[2.2, 6]$ and the range of correlation ρ is set to $[-0.4, 0.5]$. Both of these ranges are smaller than the ranges in Table 1 to exclude cases of the market taking a net short-sale position of the dirty firm.

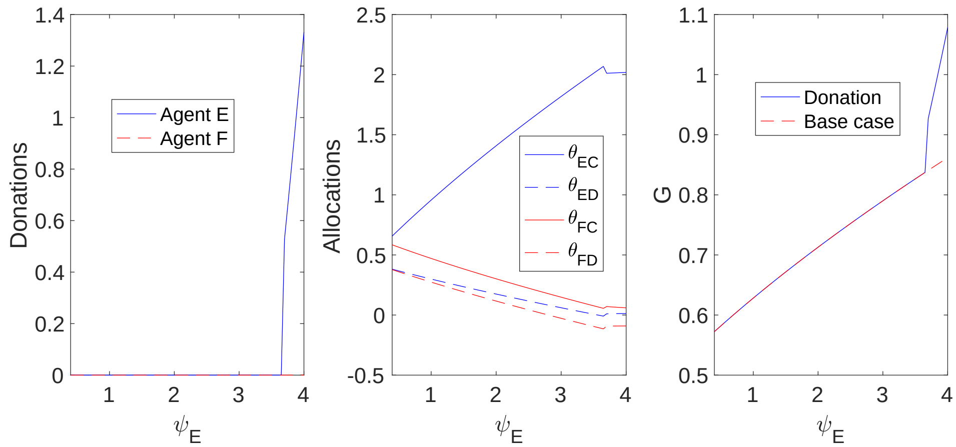

Figure 12: **Sensitivity of Donations to ψ_E.**
This Figure focuses on the economy in which investors are allowed to donate part of their wealth directly to the public good, as outlined in Section 6. It displays the level of donations (left panel), the allocations to the clean and dirty firm (middle panel) and the overall level of the public good compared to the economy without donations (right panel), as a function of parameter ψ_E, which captures the environmental investor's preference for public goods. The base values of the parameters are presented in Table 1. The range considered for parameter ψ_E has been extended to 4 in this Figure.

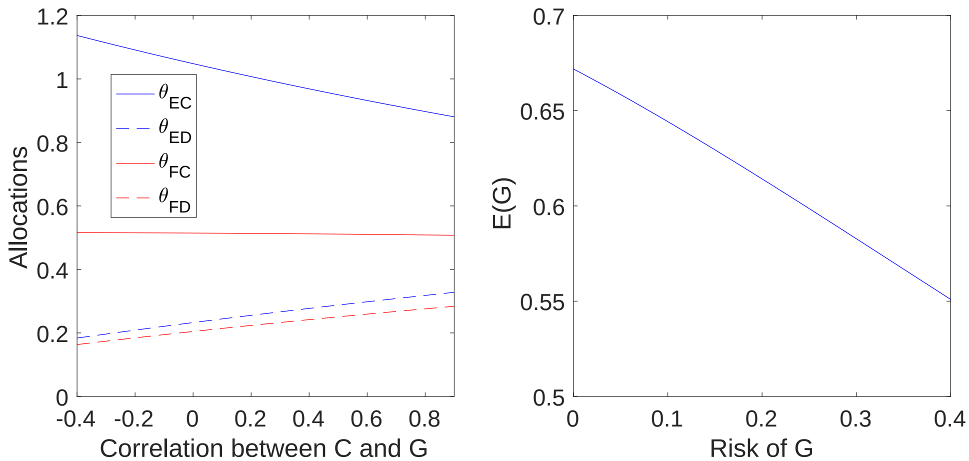

Figure 13: **Allocations and Expected Value of G with Risk**.
This Figure focuses on the economy in which there is risk in public good production, as outlined in Section 6.3. The left graph displays the sensitivity of the two investors' allocations to ρ_{CG} and the right graph displays the expected value of the public good to σ_G^2.